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CONVERSION FACTORS: U.S. CUSTOMARY ANO SI METRIC UNITS - To Convert from U.S. Customary to SI, Multiply by U.S. Units SI Metric Units inch (in.) inch (ip.) inch (in.) meter (m) centimeter (cm) millimeter (mm) 0.0254 2.54 25.4 feet (ft) yard (yd) mile (mi) meter (m) meter (m) kilometer (km) 0.305 0.914 1.609 angle, degree (°) temp., degree (°F) radian (rad) Celsius (°C) 0.0174 (°F - 32)/1.80 acre (acre) gallon (gal) hectare (ha) liter (l) 0.4052 3.79 pound (lb) ton (ton, 2000 lb) kilogram (kg) kilogram (kg) 0.4536 907.2 pound force ()bf) pound/in 2 (psi) pound/ft 2 (psf) newton (N) newton/m 2 (N/m 2) newton/m 2 (N/m 2) 4.448 6895 47.88 foot-pound (ft-lb) horsepower (hp) British thermal unit (BTU) British thermal unit (BTU) joule (J) watt (W) joule (J) kilowatt-hour (kwh) 1.356 746 1055 2.93 X 10-4 Definitions I newton-force that will give a 1-kg mass an acceleration of I m/sec 2 I joule-work done by a force of I N over a displacement of I m I pascal = I N/m 2 I kilogram force (kgf) = 9.807 N I gravity-acceleration (g) = 9.807 m/sec 2 l acre (a)= 4052 m2 l hectare (ha) = I 0,000 m2 I kip (kip)= 1000 lb Probability Concepts in Engineering Emphasis on Applications in Civil & Environmental Engineering 2NDEDITION Prob ab iIity Concepts in Engineering* Emphasis on Applications in Civil & Environmental Engineering ALFREDO H-S. LG Emeritus Professor. University of Illinois at Urbana-Champaign and University of Cal!fmia, Irvine WILSON H. TANG Chair Professor. Hong Kong University 2.1 TAB 6 7 9 10 11 TBC 2 3 2 3 2 3 2 3 2 3 CAB 1200 1200 550 550 550 CBC 300 300 300 300 300 300 300 300 300 300 TAC 8 9 9 10 11 12 12 13 13 14 CAC 1500 1500 1500 1500 850 850 850 850 850 (a) Sample space of travel time from A to B = {6, 7, 9, 10, 11} Sample space of travel time from A to C = {8, 9, 10, 11, 12, 13, 14} (b) Sample space of travel cost from A to C = {850, 1500} (c) Sample space of TAC and CAC = {(8, 1500), (9, 1500), (10, 1500), (11, 850), (12, 850), (13, 850), (14, 850)} 2.2 (a) Since the possible values of settlement for Pier 1 overlap partially with those of Pier 2, it is possible that both Piers will have the same settlement. Hence, the minimum differential settlement is zero. The maximum differential settlement will happen when the settlement of Pier 2 is 10 cm and that of Pier 1 is 2 cm, which yields a differential settlement of 8 cm. Hence, the sample space of the differential settlement is zero to 8 cm. (b) If the differential settlement is assumed to be equally likely between 0 and 8 cm, the probability that it will be between 3 and 5 cm is equal to P= 5−3 2 = = 0.25 8−0 8 2.3 (a) (b) Wind direction E1 90o E2 E3 60o E1 30o Wind Speed 0 15 35 45 (c) A and B are not mutually exclusive A and C are not mutually exclusive 2.4 (a) Inflow 6’ 6’ 6’ 7’ 7’ 7’ 8’ 8’ 8’ Possible water level Outflow Inflow – Outflow + 7’ 5’ 8’ 6’ 7’ 7’ 6’ 5’ 9’ 6’ 8’ 7’ 7’ 5’ 10’ 6’ 9’ 7’ 8’ Hence possible combinations of inflow and outflow are (6’,5’), (6’,6’), (6’,7’), (7’,5’), (7’,6’), (7’,7’), (8’,5’), (8’,6’) and (8’,7’). (b) The possible water levels in the tank are 6’, 7’, 8’, 9’ and 10’. (c) Let E = at least 9 ft of water remains in the tank at the end of day. (8’, 5’) and (8’, 6’) are favourable to the event E. So P(E) = 3 1 = . 9 3 Sample points (7’, 5’), 2.5 (a) Locations Locations Load at B Load at C MA Probability of W2 of W1 – – 0 0 0 0.15x0.2=0.03 – B 500 0 5,000 0.045 – C 0 500 10,000 0.075 B – 200 0 2,000 0.05 B B 700 0 7,000 0.075 B C 200 500 12,000 0.125 C – 0 200 4,000 0.12 C B 500 200 9,000 0.18 C C 0 700 14,000 0.30 E1 X X X X X X E2 E3 X X X X X X X X X (b) E1 and E2 are not mutually exclusive because these two events can occur together, for example, when the weight W2 is applied at C, MA is 10,000 ft-lb; hence both E1 and E2 will occur. (c) The probability of each possible value of MA is tabulated in the last column of the table above. (d) P(E1) = P(MA > 5,000) = 0.075 + 0.075 + 0.125 + 0.18 + 0.30 = 0.755 P(E2) = P(1,000 ≤ MA ≤ 12,000) = 0.045 + 0.075 + 0.05 + 0.075 + 0.12 + 0.18 = 0.545 P(E3) = 0.05 + 0.075 = 0.125 P(E1 ∩ E2) = P(5,000 < MA ≤ 12,000) = 0.075 + 0.075 + 0.18 = 0.33 P(E1 ∪ E2) = 1 – 0.03 = 0.97 P(E 2) = 0.03 + 0.125 + 0.3 = 0.455 P(P (E2) = 1 − P (E) = 1 − 0.545 = 0.455 2.6 (a) Let A1 = Lane 1 in Route A requires major surfacing A2 = Lane 2 in Route A requires major surfacing B1 = Lane 1 in Route B requires major surfacing B2 = Lane 2 in Route B requires major surfacing P(A1) = P(A2) = 0.05 P(B1) = P(B2) = 0.15 P(A2A1) = 0.15 P(B2B1) = 0.45 P(Route A will require major surfacing) = P(A) = P(A1∪A2) = P(A1) + P(A2) - P(A2A1) P(A1) = 0.05 + 0.05 - (0.15)(0.05) = 0.0925 P(Route B will require major surfacing) = P(B) = P(B1∪B2) = P(B1) + P(B2) - P(B2B1) P(B1) = 0.15 + 0.15 - (0.45)(0.15) = 0.2325 (b) P(route between cities 1 and 3 will require major resurfacing) = P(A∪B) = P(A) + P(B) - P(A)P(B) = 0.0925 + 0.2325 – (0.0925)(0.2325) = 0.302 2.7 P(Di) = 0.1 Assume condition between welds are statistically independent (a) P(D1 D 2 D3) = P(D1)P(D 2)P(D3) = 0.9 x 0.9 x 0.9 = 0.729 (b) P(Exactly two of the three welds are defective) = P(D1 D2D3 ∪ D1 D 2 D3 ∪ D1 D2 D3) = P(D1 D2D3) + P(D1 D 2 D3) + P(D1 D2 D3) since the three events are mutually exclusive. Hence, the probability become P = 0.9x0.1x0.1 + 0.1x0.9x0.1 + 0.1x0.1x0.9 = 0.027 (c) P(all 3 welds defective) = P(D1 D2 D3) = P3(D) = (0.1)3 = 0.001 2.8 P(E1) = 0.8; P(E2) = 0.7; P(E3) = 0.95 P(E3 E 2) = 0.6; assume E2 and E3 are statistically independent of E1 (a) A = (E2 ∪ E3) E1 B = (E 2 ∪ E 3) E1 or E 2 E 3 ∪ E1 (b) P(B) = P(E1 ∪ E 2 E 3) = P(E1) + P(E 2 E 3) – P(E1 E 2 E 3) = 0.2 + P(E 3 E 2)P(E 2) – P(E1)P(E 2 E 3) = 0.2 + 0.4x0.3 – 0.2x(0.4x0.3) = 0.296 (c) P(casting concrete production not feasible at site) = P((E2 ∪ E3) E1 E 2) = P(E 2 ∪ E 3) E1 E 2 P(E 2) = 0.8 x 0.6 =0.48 = P(E1 E 2 E 3) P(E 2) = P(E1) P(E 3 E 2) P(E 2) P(E 2) 2.9 E1, E2, E3 denote events tractor no. 1, 2, 3 are in good condition respectively (a) A = only tractor no. 1 is in good condition = E1 E 2 E 3 B = exactly one tractor is in good condition = E1 E 2 E 3 ∪ E1 E2 E 3 ∪ E1 E 2 E3 C = at least one tractor is in good condition = E1 ∪ E2 ∪ E3 (b) Given P(E1) = P(E2) = P(E3) = 0.6 P(E 2 E1) = 0.6 or P(E 2 E 3) = 0.6 P(E 3 E1 E 2) = 0.8 or P(E1 E 2 E 3) = 0.8 P(A) = P(E1 E 2 E 3) = P(E1 E 2 E 3) P(E 2 E 3) = [1 – P(E1 E 2 E 3)] P(E 2 E 3) P(E 3) = (1 - 0.8)(0.6)(0.4) = 0.048 Since E1 E 2 E 3 , E1 E2 E 3 and E1 E 2 E3 are mutually exclusive; also the probability of each of these three events is the same, P(B) = 3 x P(E1 E 2 E 3) = 3 x 0.048 = 0.144 P(C) = P(E1 ∪ E2 ∪ E3) = 1 – P(E1 ∪ E 2 ∪ E 3) = 1 – P(E1 E 2 E 3) = 1 – P(E1 E 2 E 3) P(E 2 E 3) P(E 3) = 1 – 0.8x0.6x0.4 = 0.808 2.10 (a) The event “both subcontractors will be available” = AB, hence since P(A∪B) = P(A) + P(B) - P(AB) ⇒ P(AB) = P(A) + P(B) - P(A∪B) = 0.6 + 0.8 - 0.9 = 0.5 (b) P(B is availableA is not available) = P(B A) = P(BA) P(A) while it is clear from the following Venn diagram that P(BA) = P(B) - P(AB). A AB B A Hence P(BA) P(B) − P(AB) = 1 − P(A) P(A) = (0.8 - 0.5)/(1 - 0.6) = 0.3/0.4 = 0.75 (c) (i) If A and B are s.i., we must have P(BA) = P(B) = 0.8. However, using Bayes’ rule, P(BA) = P(AB)/P(A) = 0.5/0.6 = 0.8333 So A and B are not s.i. (A’s being available boosts the chances that B will be available) (ii) From (a), P(AB) is nonzero, hence AB ≠ ∅, i.e. A and B are not m.e. (iii) Given: P(A∪B) = 0.9 ⇒ A∪B does not generate the whole sample space (otherwise the probability would be 1), i.e. A and B are not collectively exhaustive. 2.11 (a) Let L, SA, SB denote the respective events “leakage at site”, “seam of sand from X to A”, “seam of sand from X to B”. Given probabilities: P(L) = 0.01, P(SA) = 0.02, P(SB) = 0.03, P(SBSA) = 0.2, Also given: independence between leakage and seams of sand, i.e. P(SBL) = P(SB), P(LSB) = P(L), P(SAL) = P(SA), P(L SA) = P(L) The event “water in town A will be contaminated” = L∩SA, whose probability is P(L∩SA) = P(L SA) P(SA) = P(L)P(SA) = 0.01×0.02 = 0.0002. (b) The desired event is (L SA) ∪ (L SB), whose probability is P(L SA) + P(L SB) - P(L SA L SB) = P(L)P(SA) + P(L)P(SB) - P(L SA SB) = P(L) [P(SA) + P(SB) - P(SA SB)] = P(L) [P(SA) + P(SB) - P(SBSA) P(SA)] = 0.01 (0.02 + 0.03 - 0.2×0.02) = 0.00046 2.12 Let A,B,C denote the respective events that the named towns are flooded. Given probabilities: P(A) = 0.2, P(B) = 0.3, P(C) = 0.1, P(B | C) = 0.6, P(A | BC) = 0.8, P(AB | C) = 0.9, where an overbar denotes compliment of an event. (a) P(disaster year) = P(ABC) = P(A | BC)P(BC) = P(A | BC)P(B | C)P(C) = 0.8×0.6×0.1 = 0.048 (b) P(C | B) = P(BC) / P(B) = P(B | C)P(C) / P(B) = 0.6×0.1÷0.3 = 0.2 (c) The event of interest is A∪B∪C. Since this is the union of many items, we can work with its compliment instead, allowing us to apply De Morgan’s rule and rewrite as P(A∪B∪C) = 1 – P(A ∪ B ∪ C) = 1 – P(ABC) = 1 – P(AB | C)P(C) = 1 – 0.9×(1 – 0.1) = 1 – 0.81 = 0.19 by De Morgan’s rule, 2.13 [(L∪M) G]] ∪[(LM) G] (a) C= (b) Since (L∪M)G is contained in G, it is mutually exclusive to (LM) G which is contained in G . Hence P(C) is simply the sum of two terms, P(C) = P[(L∪M) G]] + P[(L∩M) G] = P(LG∪MG) + P(L)P(M G) = P(LG) + P(MG) – P(LMG) + P(L)P(M| G)P(G) = P(L)P(G) + P(M|G)P(G) – P(L)P(M|G)P(G)+ P(L)P(M| G)P(G) = 0.7×0.6 + 1×0.6 – 0.7×1×0.6 + 0.7×0.5×0.4 = 0.74 (c) P(L |C) = P(L C)/P(C) = P(L {[(L∪M) G]} ∪[(LM) G])) / P(C) = P[L (L∪M) G]] / P(C) impossible event = P(L MG) / P(C) = P(L)P(MG) / P(C) = P(L)P(M|G)P(G) / P(C) = 0.3×1×0.6 / 0.74 ≅ 0.243 since (L L)M G is an 2.14 (a) Note that the question assumes an accident either occurs or does not occur at a given crossing each year, i.e. no more than one accident per year. There were a total of 30 + 20 + 60 + 20 = 130 accidents in 10 years, hence the yearly average is 130 / 10 = 13 accidents among 1000 crossings, hence the yearly probability of accident occurring at a given crossing is 13 = 0.013 (probability per year) 1000 (b) Examining the data across the “Day” row, we see that the relative likelihood of R compared to S is 30:60, hence P(S | D) = 60/90 = 2/3 (c) Let F denote “fatal accident”. We have 50% of (30 + 20) = 0.5×50 = 25 fatal “run into train” accidents, and 80% of (60 + 20) = 0.8×80 = 64 fatal “struck by train” accidents, hence the total is P(F) = (25 + 64) / 130 ≅ 0.685 (d) (i) (ii) D and R are not mutually exclusive; they can occur together (there were 30 run-into-train accidents happened in daytime); If D and R are, we must have P(R | D) = P(R), but here P(R | D) = 30 / 90 = 1/3, while P(R) = (30 + 20) / 130 = 5/13, so D and R are not statistically independent. 2.15 F = fuel cell technology successfully marketable S = solar power technology successfully marketable F and S are statistically independent Given P(F) = 0.7; P(S) = 0.85 (i) P(energy supplied) = P(F∪S) = P(F) + P(S) – P(F)P(S) = 0.7 + 0.85 – 0.7x0.85 = 0.955 (ii) P(only one source of energy available) = P(F S ∪ F S) = P(F S) + P(F S) = (0.7)(1-0.85) + (1-0.7)(0.85) = 0.36 2.16 a. E1 = Monday is a rainy day E2 = Tuesday is a rainy day E3 = Wednesday is a rainy day Given P(E1) = P(E2) = P(E3) = 0.3 P(E2 E1) = P(E3 E2) = 0.5 P(E3 E1E2) = 0.2 b. P(E1E2) = P(E2 E1) P(E1) = 0.5x0.3 = 0.15 c. P(E1E2 E 3) = P(E 3 E1E2) P(E2 E1) P(E1) = (1-0.2)(0.5)(0.3) = 0.12 d. P(at least one rainy day) = P(E1 ∪ E2 ∪ E3) = P(E1) + P(E2) + P(E3) - P(E1E2) - P(E2E3) - P(E1E3) + P(E1E2E3) = 0.3 + 0.3 + 0.3 - 0.3x0.5 - 0.1125 + 0.3x0.5x0.2 =0.52 Where P(E1E3)=P(E3|E1)P(E1)=0.375x0.3=0.1125 P(E3|E1)= P(E3|E2E1)P(E2|E1)+ P(E3| E 2 E1)P(E 2 |E1) =0.15x0.15+0.3x0.5 =0.375 2.17 Let A, B, C denote events parking lot A, B and C are available on a week day morning respectively Given: P(A) = 0.2; P(B) = 0.15; P(C) = 0.8 P(B A) = 0.5; P(C A B) = 0.4 (a) P(no free parking) = P(A B) = P(A) x P(B A) = 0.8 x 0.5 = 0.4 (b) P(able to park) = 1 – P(not able to park) = 1 – P(A B C) = 1 – P(A B)xP(C A B) = 1 – 0.4x(1-0.4) = 1 – 0.24 =0.76 (c) P(free parkingable to park) P(A∪BA∪B∪C) = P[(A ∪ B)(A ∪ B ∪ C)] P(A ∪ B ∪ C) = P(A ∪ B) P(A ∪ B ∪ C) = 1 − P(AB) 0.76 = 1 − 0.4 = 0.789 0.76 2.18 C = Collapse of superstructure E = Excessive settlement Given: P(E) = 0.1; P(C) = 0.05 P(CE) = 0.2 (a) P(Failure) = P(C ∪ E) = P(C) + P(E) – P(CE)P(E) = 0.05 + 0.1 – 0.2x0.1 = 0.13 (b) P(ECE ∪ C) = P((EC) ∩ (E ∪ C)) P(E ∪ C) = P(EC) / 0.13 = 0.2x0.1 / 0.13 = 0.154 (c) P(E C ∪ E C) = P(E C) + P(E C) = P(E∪C) – P(EC) = 0.13 – 0.02 =0.11 2.19 M = failure of master cylinder W = failure of wheel cylinders B = failure of brake pads Given: P(M) = 0.02; P(W) = 0.05; P(B) = 0.5 P(MW) = 0.01 B is statistically independent of M or W (a) P(W M B) = P(W M)P(B) = [P(W) – P(MW)] [1 – P(B)] = (0.05 – 0.01)(1 – 0.5) = 0.02 (b) P(system failure) = 1 – P(no component failure) = 1- P(M W B) = 1- P(M W) P(B) = 1- {1 - P(M ∪ W)}P(B) = 1- {1-[P(M)+P(W)-P(MW)]}P(B) = {1 - [0.02+0.05-0.01]}(0.5) = 0.53 (c) P(only one component failed) = P(W M B) + P(W M B) + P(W M B) P(W M B) = 0.02 from part (a) also P(W M B) = P(W M) x P(B) = [P(M)-P(MW)]P(B) = (0.02 – 0.01)x0.5 = 0.005 also P(W M B) = P(W M) x P(B) = [1-P(M)-P(W)+P(MW)]P(B) = (1-0.02-0.005+0.01)x0.5 = 0.47 Since the three events W M B, W M B and W M B are all within the event of system failure P(only one component failuresystem failure) = (0.47+0.005+0.02) / 0.53 = 0.934 2.20 E1 = Excessive snowfall in first winter E2 = Excessive snowfall in second winter E3 = Excessive snowfall in third winter (a) P(E1) = P(E2) = P(E3) = 0.1 P(E2 E1) = 0.4 = P(E3 E2) P(E3 E1 E2) = 0.2 (b) P(E1 ∪ E2) = P(E1)+P(E2)-P(E1 E2) = 0.1 + 0.1 – 0.4x0.1 = 0.16 (c) P(E1E2E3) = P(E1) P(E2 E1) P(E3 E1 E2) = 0.1x0.4x0.2 =0.008 (d) P(E2 E1) = ? Since P(E1 ∪ E2) = P(E1)+P(E2)-P(E1 E2) = P(E1)+P(E2)-P(E2 E1)P(E1) = 0.9+0.9-P(E2 E1) 0.9 and from given relationship, P(E1 ∪ E2) = 1- P(E1E2) = 1- P(E1) P(E2 E1) = 1-0.1x0.4 = 0.96 Therefore, 1.8-0.9 P(E2 E1) = 0.96 and P(E2 E1) = 0.933 2.21 H1, H2, H3 denote first, second and third summer is hot respectively Given: P(H1) = P(H2) = P(H3) = 0.2 P(H2 H1) = P(H3 E2) = 0.4 (a) P(H1H2H3) = P(H1) P(H2 H1) P(H3 E2) = 0.2x0.4x0.4 = 0.032 (b) P(H 2 H 1) = ? Using the hint given in part (d) of P2.20, P(H 1) + P(H 2) - P(H 2 H 1)P(H 1) = P(H 1 ∪ H 2) = 1 – P(H1H2) = 1 - P(H2 H1) P(H1) we have, 0.8 + 0.8 - P(H 2 H 1)(0.8) = 1 – 0.4x0.2 therefore, P(H 2 H 1) = 0.85 (c) P(at least 1 hot summer) = 1 – P(no hot summers) = 1 – P(H 1 H 2 H 3) = 1 – P(H 1)P(H 2 H 1) P(H 3 H 2) = 1 – 0.8x0.85x0.85 = 0.422 2.22 A = Shut down of Plant A B = Shut down of Plant B C = Shut down of Plant C Given: P(A) = 0.05, P(B) = 0.05, P(C) = 0.1 P(BA) = 0.5 = P(AB) C is statistically independent of A and B (a) P(complete backoutA) = P(BCA) = P(BA)P(C) = 0.5x0.1 = 0.05 (b) P(no power) = P(ABC) = P(AB)P(C) = P(BA)P(A)P(C) = 0.5x0.05x0.1 =0.00025 (c) P(less than or equal to 100 MW capacity) = P (at most two plants operating) = 1 – P(all plants operating) = 1 – P(A B C) = 1 – P(C)P(A B) = 1 – P(C) [1- {P(A)+P(B)- P(BA)P(A)}] = 1 – 0.9[1-{0.05+0.05-0.5x0.05}] = 0.1675 2.23 P(Damage) = P(D) = 0.02 Assume damages between earth quakes are statistically independent (a) P(no damage in all three earthquakes) = P3(D) = 0.023 = 8x10-6 (b) P(D1 D2) = P(D1)P(D2) = 0.98x0.02 = 0.0196 2.24 (a) Let A, B denote the event of the respective engineers spotting the error. Let E denote the event that the error is spotted, P(E) = P(A∪B) = P(A) + P(B) – P(AB) = P(A) + P(B) – P(A)P(B) = 0.8 + 0.9 – 0.8×0.9 = 0.98 (b) “Spotted by A alone” implies that B failed to spot it, hence the required probability is P(AB’|E) = P(AB’∩E)/P(E) = P(E|AB’)P(AB’) / P(E) = 1×P(A)P(B’) / P(E) = 0.8×0.1 / 0.98 ≅ 0.082 (c) With these 3 engineers checking it, the probability of not finding the error is P(C1’C2’C3’) = P(C1’)P(C2’)P(C3’) = (1 – 0.75)3 , hence P(error spotted) = 1 – (1 – 0.75)3 ≅ 0.984, which is higher than the 0.98 in (a), so the team of 3 is better. (d) The probability that the first error is detected (event D1) has been calculated in (a) to be 0.98. However, since statistical independence is given, detection of the second error (event D2) 2 still has the same probability. Hence P(D1 D2) = P(D1)P(D2) = 0.98 ≅ 0.960 2.25 L = Failure of lattice structure A = Failure of anchorage Given: P(A) = 0.006 P(LA) = 0.4 P(AL) = 0.3 Hence, P(L) = P(LA)P(A) / P(AL) = 0.008 (a) P(antenna disk damage) = P(A∪L) = P(A) + P(L) – P(A)P(LA) = 0.4 + 0.008 - 0.006x0.4 = 0.406 (b) P(only one of the two potential failure modes) = P(A L) + P(A L) = P(L A)P(A) + P(A L)P(L)(0.008) = (1-0.4)(0.006) + (1-0.3)(0.008) = 0.0036 + 0.0056 =0.0092 (c) P (A D) = P(A(A ∪ L)) P(A) 0.006 = = = 0.0148 P(A ∪ L) P (A ∪ L) 0.406 2.26 P(F) = 0.01 P(A F) = 0.1 P(AF) = 1 (a) FA, F A , F A, F A are set of mutually exclusive and collectively exhaustive events (b) P(FA) = P(AF)P(F) = 1x0.01 = 0.01 P(F A) = P(A F)P(F) = 0x0.01 = 0 P(F A) = P(A F)P(F) = 0.1x0.99 = 0.099 P(F A) = P(A F)P(F) = 0.9x0.99 = 0.891 (c) P(A) = P(AF)P(F) + P(A F)P(F) = 1x0.01 + 0.1x0.99 = 0.109 (d) P(FA) = P(A F) P(F) P(A) = 0.01 = 0.0917 0.109 2.27 Given: P(D) = 0.001 P(TD) = 0.85; P(T D) = 0.02 (a) DT, D T , D T, D T (b) P(DT) = P(TD)P(D) = 0.85x0.001 = 0.00085 P(D T) = P(T D)P(D) = 0.15x0.001 = 0.00015 P(D T) = P(T D)P(D) = 0.02x0.999 = 0.01998 P(D T) = P(T D)P(D) = 0.98x0.999 = 0.979 (c) P(D T) = P(T D) P(D) P(T D) P(D) + P(T D) P(D) = 0.00085 = 0.0408 0.00085 + 0.01998 2.28 Given: P(RA) = P(GA) = 0.5 P(RB) = P(GB) = 0.05 P(GBGA) = 0.8 P(GLT) = 0.2 Signal at C is statistically independent of those at A or B. (a) E1 = RA∪ RB E2 = GLT E3 = RA GB ∪ GA RB (b) P(stopped at least once from M to Q) = 1 – P(no stopping at all from M to Q) = 1 – P(GA GB GLT) = 1 – P(GLT)P(GA)P(GB GA) = 1 – 0.2x0.5x0.8 = 0.92 (c) P(stopped at most once from M to N) = 1 – P(stopped at both A and B) = 1 – P(RARB) = 1 - P(RA) P(RBRA) From the hint given in P2.20, it can be shown that P(RB R A) = 1 1 [1 − P(G B G A) P(G A)] − 1 = [1 − 0.8 × 0.5] − 1 = 0.2 P(R A) 0 .5 Hence P(stopped at most once from M to N) = 1 – 0.5x0.2 = 0.9 2.29 (a) P(WSC) = P(W)P(SC) = P(W)P(S|C)P(C) = 0.1×0.3×0.05 = 0.0015 (b) P(W∪C) = P(W) + P(C) – P(WC) = P(W) + P(C) – P(W)P(C) = 0.1 + 0.05 – 0.1×0.05 = 0.145 P(W∪C| S) = P[S (W∪C)] / P(S) = P(S W∪ S C) / P(S) = [P(S W) + P(S C) – P(S WC)] / P(S) = [P(S)P(W) + P(S |C)P(C) – P(W)P(S C)] / P(S) = [0.8×0.1 + (1 – 0.3)×0.05 – 0.1×(1 – 0.3)×0.05] / (1 – 0.2) ≅ 0.139 (c) (d) P(nice winter day) = P(S W C) = P(W)P(S C) = P(W)P(S | C)P(C) = 0.9x0.7x0.05 = 0.0315 (e) P(U) = P(U |CW)P(CW) + P(U | C W)P(C W) + P(U|C W) P(C| W) + P(U|| C W) P(C W) = 1xP(W)P(C) + 0.5xP(W)P(C) + 0.5xP(C)P(W) + 0xP(C)P(W) = 1x0.1x0.05 + 0.5x0.1x0.95 + 0.5x0.05x0.9 = 0.075 2.30 (a) Let L and B denote the respective events of lead and bacteria contamination, and C denote water contamination. P(C) = P(L∪B) = P(L) + P(B) – P(LB) = P(L) + P(B) – P(L)P(B)∵ L and B are independent events = 0.04 + 0.02 – 0.04×0.02 = 0.0592 (b) P(L B | C) = P(CL B) / P(C) = P(C | L B)P(L B) / P(C), but P(C | L B) = 1 (∵ lead alone will contaminate for sure) and P(L B) = P(L)P(B) (∵ statistical independence), hence the probability = 1×P(L)P(B) / P(C) = 1×0.04×(1 – 0.02) / 0.0592 ≅ 0.662 2.31 C 0.1 0.1 0.1 0.2 0.2 0.2 Water level 0 10 20 0 10 20 W (x103 lb) 100 210 320 100 210 320 F (x103 lb) 10 21 32 20 42 64 (a) Sample space of F = {10, 20, 21, 32, 42, 64} (b) Let H be the horizontal force in 103 lb P(sliding) = P(H>F) = P(F 40) ÷ P(X > 0) Let's first calculate P(X > 40): comparing similar triangles formed by the PDF and the x-axis (with vertical edges at x = 10 and at x = 40, respectively), we see that P(X > 40) = Area of smaller triangle 2 ⎛ 70 − 40 ⎞ =⎜ ⎟ × Area of larger triangle ⎝ 70 − 10 ⎠ = 0.52 [(70 - 10)×(0.02) ÷ 2] = 0.015 Hence the required probability P(X > 40 | X > 0) is 0.015 ÷ [10×0.02 + (70 - 10)×(0.02) ÷ 2] = 0.015 ÷ 0.08 = 0.1875 3.3 ∞ (a) Applying the normalization condition ∫f X (x)dx = 1 −∞ 6 ⎡ x2 x3 ⎤ x2 ⇒ c(x −)dx = 1 ⇒ c ⎢ − ⎥ = 1 6 ⎣ 2 18 ⎦ 0 0 18 = 1/6 ⇒ c= 9 × 36 − 6 3 6 ∫ (b) To avoid repeating integration, let’s work with the CDF of X, which is 1 ⎡ x2 x3 ⎤ FX(x) = ⎢ − ⎥ 6 ⎣ 2 18 ⎦ 2 3 = (9x – x)/108 (for x between 0 and 6 only) Since overflow already occurred, the given event is X > 4 (cms), hence the conditional probability P(X < 5 | X > 4) = P(X < 5 and X > 4) / P(X > 4) = P(4 < X < 5) / [1 – P(X ≤ 4)] = [FX(5) - FX(4)] / [1 - FX(4)] 2 3 2 3 2 3 = [(9×5 – 5) – (9×4 – 4)]/ [108 – (9×4 – 4)] = (100 – 80) / (108 – 80) = 20/28 = 5/7 ≅ 0.714 (c) Let C denote “completion of pipe replacement by the next storm”, where P(C) = 0.6. If C indeed occurs, overflow means X > 5, whereas if C did not occur then overflow would correspond to X > 4. Hence the total probability of overflow is (with ’ denoting compliment) P(overflow) = P(overflow | C)P(C) + P(overflow | C’)P(C’) = P(X > 5)×0.6 + P(X > 4)×(1 – 0.6) = [1 - FX(5)]×0.6 + [1 - FX(4)]×0.4 = (1 – 100/108)×0.6 + (1 – 80/108)×0.4 ≅ 0.148 3.4 (a) The median of X, xm, is obtained by solving the equation that defines xm, P(X ≤ xm) = 0.5 ⇒ FX(xm) = 0.5 ⇒ 1- (10 ÷ xm)4 = 0.5 ⇒ xm ≈11.9 (inches) (b) First, obtain the PDF of X: fX(x) = dFX/dx = 4×104x-5 for x ≥ 10, and zero elsewhere. Hence the expected amount of snowfall in a severe snow storm is ∞ E(X) = ∫ xf X (x)dx −∞ ∞ = ∫ 4 × 10 4 x − 4 dx 10 = − 4 4 −3 ∞ 10 [x] 10 3 4 = × 10 4 ≈ 13.3 (inches) 3 (c) P(disastrous severe snowstorm) = P(X > 15) = 1 - P(X ≤ 15) = 1 - [1 - (10/15)4] = (2/3)4 ≈ 0.2 ∴About 20% of snow storms are disastrous. (d) Let N denote "No disastrous snow storm in a given year". Also, let E0, E1, E2 denote the respective events of experiencing 0,1,2 severe snow storms in a year. In each event, the (conditional) probability of N can be computed: P(N | E0) = 1 (if there's no severe snow storm to begin with, definitely there won't be any disastrous one); P(N | E1) = P(X ≤ 15) = 1 - (10/15)4 = 1 - (2/3)4; P(N | E2) = [P(X ≤ 15)]2 = [1 - (2/3)4]2 (statistical independence between storms); Noting that E0, E1 and E2 are m.e. and c.e., the total probability of N can be computed: P(N) = P(N | E0)P(E0) + P(N | E1)P(E1) + P(N | E2)P(E2) = 1×0.5 + [1 - (2/3)4]×0.4 + [1 - (2/3)4]2×0.1 ≈ 0.885 3.5 Let F be the final cost (a random variable), and C be the estimated cost (a constant), hence X=F/C is a random variable. (a) To satisfy the normalization condition, a ∫ 1 a 3 3 ⎡ − 3⎤ dx = ⎢ ⎥ = 3 − = 1 , 2 a x ⎣ x ⎦1 hence a = 3/2 =1.5 (b) The given event is “F exceeds C by more than 25%”, i.e. “F > 1.25C”, i.e. “F / C > 1.25”, whose probability is ∞ P(X > 1.25) = ∫f X (x) dx 1.25 1.5 1.5 3 ⎡ − 3⎤ dx = ⎢ ⎥ 2 x ⎣ x ⎦ 1.25 1.25 = -2 – (- 2.4) = 0.4 = ∫ (c) The mean, 1.5 E(X) = 3 ∫x x 2 dx = [3 ln x]1 ≅ 1.216, 1.5 1 while 1.5 2 E(X) = ∫x 1 2 3 dx = 3(1.5 – 1) = 1.5, x2 with these, we can determine the variance 2 2 Var(X) = E(X) – [E(X)] 2 = 1.5 – 1.216395324 = 0.020382415 ∴σX = 0.020382415 ≅ 0.143 3.6 ∞ 2 8 2 x x3 (a) E(X) = ∫x f X (x)dx = x dx + x 2 dx = 8 24 x −∞ 0 2 ∫ ∫ 2 ∫ 8 + 2 ln x 2 = 1/3 + 2(ln 8 – ln 2) ≅ 3.106 0 3 (b) P(X < 3 | X > 2) = 1– 2/3 = 4/9 1 – 1/ 4 P(X < 3 and X > 2) P(2 < X < 3) = P (X > 2) P (X > 2) ∫ 2 2 dx x2 2 x 1− dx 8 0 ∫ − = 2 x 3 2 2 ⎡x ⎤ 1− ⎢ ⎥ 16 ⎦ 0 2 = 3.7 (a) The mean and median of X are 13.3 lb/ft2 and 11.9 lb/ft2, respectively (as done in Problem 3-3-3). (b) The event “roof failure in a given year” means that the annual maximum snow load exceeds the design value, i.e. X > 30, whose probability is P(X > 30) = 1 – P(X ≤ 30) = 1 - FX(30) 4 = 1 - [1 - (10/30)] 4 = (1/3) = 1/81 ≅ 0.0123 ≡ p Now for the first failure to occur in the 5th year, there must be four years of non-failure followed by one failure, and the probability of such an event is 4 4 (1 – p)4p = [1 - (3/4)4] ×(1/3) ≅ 0.0117 (b) Among the next 10 years, let Y count the number of years in which failure occurs. Y follows a binomial distribution with n = 10 and p = 1/81, hence the desired probability is P(Y < 2) = P(Y = 0) + P(Y = 1) n n–1 = (1 – p) + n(1 – p) p = (80/81)10 + 10×(80/81)9×(1/81) ≅ 0.994 3.8 (a) Note that we are given the CDF, FX(x) = P(X ≤ x) and it is a continuous function. Hence P(2 ≤ X ≤ 8) = FX(8) – FX(2) = 0.8 – 0.4 = 0.4 (b) The median is defined by the particular x value where F(x) = 0.5, which occurs somewhere along x = 2 and x = 6; precisely, the constant slope there gives (0.5 – 0.4) / (xm – 2) = (0.6 – 0.4) / (6 – 2) ⇒ xm = 2 + 2 = 4 (c) The PDF is the derivative (i.e. slope) of the CDF, hence it is the following piecewise constant function: f (x) X 0.2 0.1 0.05 0.05 2 6 10 x (d) From the CDF, it is seen that one may view X as a discrete R.V. which takes on the values 1, 4, 8 with respective probabilities 0.4, 0.2, 0.4 (recall probability = area of each strip) for calculating E(X). Hence E(X) = 1×0.4 + 4×0.2 + 8×0.4 = 4.4 (integration would give the same result) 3.9 (a) Differentiating the CDF gives the PDF, 0 for s ≤ 0 ⎧ ⎪ s2 s + for 0 < s ≤ 12 f S (s) = ⎨ ⎪ 288 24 0 for s > 12 ⎩ s is where fS has a maximum, hence setting its derivative to The mode ~ s)=0 fS’(~ ~ ⇒ - s /144 + 1/24 = 0 ⇒ the mode ~ s = 6. The mean of S, ∞ E(S) = ∫ 12 ∫ sf (s) ds = (s = −∞ 0 = – s3 s2 +)ds 288 24 – (12 4) 12 3 + = = - 18 + 24 = 6 4 × 288 3 × 24 (b) Dividing the sample space into two regions R = 10 and R = 13, the total probability of failure is P(S > R) = P(S > R | R = 10)P(R = 10) + P(S > R)P(R = 13) = [1 – FS(10)]×0.7 + [1 – FS(13)]×0.3 3 2 = [1 – (-10 / 864 + 10 / 48)]×0.7 + [1 – 1]×0.3 = 0.074074074×0.7 ≅ 0.0519 3.10 (a) The only region where fX(x) is non-zero is between x = 0 and x = 20, where fX(x) = F’X(x) = - 0.005x + 0.1 which is a straight line segment decreasing from y = 0.1 (at x = 0) to y = 0 (at x = 20) fX(x) x 0 xm 2 (b) The median divides the triangular area under fX into two equal parts, hence, comparing the two similar triangles which have area ratio 2:1, one must have 0.5 (20 – xm) / 20 = (1 / 2) 0.5 ⇒ xm = 20(1 – 0.5) ≅ 5.858 3.11 x5 0 ⎧ ⎪ x 2 / 16 ⎪ (a) Integration of the PDF gives FX (x) = ⎨ ⎪ x / 4 − 1/4 ⎪ ⎩ 1 fX(x) 1/ 1 FX(x) (2,1/ x 0 5 2 x 0 2 2 5 5 5 ⎡ x2 ⎤ ⎡ x3 ⎤ x(x / 8)dx + x(1 / 4)dx = ⎢ ⎥ + ⎢ ⎥ = 71 / 24 ≅ 2.96 (mm) ⎣ 24 ⎦ 0 ⎣ 8 ⎦ 2 0 2 (c) P(X < 4) = 1 - P(X > 4) Where P(X > 4) is easily read off from the PDF as the area (5 – 4)(1/4) = 1/4, hence P(X < 4) = 1 – 1/4 = 3/4 = 0.75 2 (b) E(X) = ∫ ∫ (d) A vertical line drawn at the median xm would divide the unit area under fX into two equal halves; the right hand rectangle having area 0.5 = (5 – xm)(1/4) ⇒ xm = 5 – 4(0.5) = 3 (mm) (e) Since each of the four cracks has p = 0.25 probability of exceeding 4mm (as calculated in (c)), only one of them exceeding 4mm has the binomial probability (where n = 4, p = 0.25) 3 4×0.75 ×0.25 ≅ 0.422 3.12 x (a) For 3 ≤ x ≤ 6, FX(x) = 24 ∫t 3 dt = 4 / 3 − 12 / x 2 ; elsewhere FX is either 0 (for x < 3) or 1 (for x > 3 6) FX is a parabola going from x = 3 to x = 6, followed by a horizontal line when plotted 6 (b) E(X) = 24 ∫ x(x 3)dx = 24(1/3 – 1/6) = 4 (tons) 3 6 2 2 (c) First calculated the variance, Var(X) = E(X) – [E(X)] 2 = ∫x 2 (3 24)dx - 16 = 24(ln 6 – ln 3) = x3 24 ln 2 - 16 = c.o.v. = σ 24 ln 2 − 16 = ≅ 19.9% μ 4 (d) When the load X exceeds 5.5 tons, the roof will collapse, hence P(roof collapse) = P(X > 5.5) = 1 – P(X ≤ 5.5) 2 = 1 – FX(5.5) = 1 – (4/3 – 12/5.5) ≅ 0.063 3.13 (a) Since the return period, τ, is 200 years, this means the yearly probability of exceedance (i.e. encountering waves exceeding the design value) is p = 1 / τ = 1 / (200 years) = 0.005 (probability per year) (b) For each year, the probability of non-exceedance is 1 – 0.005 = 0.995, while the intended lifetime is 30 years. Hence non-exceedance during the whole lifetime has the binomial probability, 0.995 ≅ 0.860 30 3.14 Design life = 50 years Mean rate of high intensity earthquake = 1/100 = 0.01/yr P(no damage within 50 years) = 0.99 Let P = probability of damage under a single earthquake (a) Using a Bernoulli Sequence Model for occurrence of high intensity earthquakes, P(quake each year) = 1/100 = 0.01 P(damage each year) = 0.01p P(no damage in 50 years) = (1 – 0.01p)50 ≅ 0.99 Hence, 1 – 0.01p = (0.99)1/50 = (0.99)0.02 = 0.9998 or p = 0.02 (b) Assuming a Poisson process for the quake occurrence, Mean rate of damaging earthquake = 0.01p = 0.01x0.02 = 0.0002/yr P(damage in 20 years) = 1 – P(no damage in 20 years) = 1 – P(no occurrence of damaging quakes in 20 years) = 1 - e-0.0002x20 = 1 – e-0.004 = 0.004 3.15 Failure rate = ν = 1/5000 = 0.0002 per hour (a) P(no failure between inspection) = P(N=0 in 2500 hr) (0.0002 × 2500) 0 e −0.0002×2500 = = e-0.5 = 0.607 0! Hence P(failure between inspection) = 1 – 0.607 = 0.393 (b) P(at most two failed aircrafts among 10) = P(M=0)-P(M=1)-P(M=2) = 10! 10! 10! (0.393)0(0.607)10 (0.393)(0.607)9 (0.393)2(0.607)8 0!× 10! 1!× 9! 2!× 8! = 0.0068 – 0.044 – 0.1281 = 0.179 (c) Let t be the revised inspection/maintenance interval P(failure between inspection) = 1-P(no failure between inspection) = 1-e-0.0002t = 0.05 -0.0002t = ln 0.95 and t = 256.5 → 257 hours 3.16 (a) Let C be the number of microscopic cracks along a 20-feet beam. C has a Poisson distribution with mean rate νC = 1/40 (number per foot), and length of observation t = 20 feet, hence the parameter λ = (1/40) (20) = 0.5, thus P(C = 2) = e-0.5 (0.52 / 2!) ≅ 0.076 (b) Let S denote the number of slag inclusions along a 20-feet beam. S has a Poisson distribution with mean rate νS = 1/25 (number per foot), and length of observation t = 20 feet, hence the parameter λ = (1/25)(20) = 0.8, thus 1 – P(S = 0) = 1 - e-0.8 ≅ 0.551 (c) Let X be the total number of flaws along a 20-feet beam. Along 1000 feet (say) of such a beam, one can expect (1/40)(1000) = 25 cracks and (1/25)(1000) = 40 slag inclusions. Hence the mean rate of flaw would be ν = (25 + 40)/1000 or simply (1/40) + (1/25) = 0.065 flaws per foot, which is multiplied to the length of observation, t = 20 feet to get the parameter λ = 1.3. Hence P(X > 2) = 1 – P(X ≤ 2) = 1 – e ≅ 0.143 -1.3 2 (1 + 1.3 + 1.3 / 2!) = 1 - 0.857112489 (d) Thinking of beam rejection as “success”, the total number (N) of beams rejected among 4 would follow a binomial distribution with n = 4 and p = answer in part (c). Thus ⎛ 4⎞ P(N = 1) = ⎜⎜ ⎟⎟ 4×0.142887511×0.8573 ≅ 0.360 ⎝1⎠ 3.17 (a) Let N be the number of poor air quality periods during the next 4.5 months; N follows a Poisson process with mean value (1/month)(4.5 months) = 4.5, hence -4.5 2 P(N ≤ 2) = e (1 + 4.5 + 4.5 /2!) ≅ 0.174 (b) Since only 10% of poor quality periods have hazardous levels, the “hazardous” periods (H) must occur at a mean rate of 1 per month×10% = 0.1 per month, hence, over 3 months, H has the mean λH = (0.1)(3) = 0.3 -0.3 ≅ 0.259 ⇒ P(ever hazardous) = 1 – P(H = 0) = 1 - e Alternative approach: use total probability theorem: although there is (1 – 0.1) = 0.9 probability of non-hazardous pollution level during a poor air quality period, during a 3month period there could be any number (n) such periods and the probability of nonn hazardous level reduces to 0.9 for a given n. Hence the total probability of non-hazardous level during the whole time is ∞ ∞ ∞ e – (1×3) (1 × 3) n (3 × 0.9) n -3 3×0.9 -0.3 =e e 0.9 n P (N = n) = = e , hence 0.9 n = e −3 n! n! n =0 n =0 n =0 -0.3 P(ever hazardous) = 1 – P(never hazardous) = 1 – e = 1 - 0.741 ≅ 0.259 ∑ ∑ ∑ 3.18 (a) Let E and T denote the number of earthquakes and tornadoes in one year, respectively. They are both Poisson random variables with respective means 1 λE = νEt = ×1 year = 0.1; λT = 0.3 10 years Also, the (yearly) probability of flooding, P(F) = 1/5 = 0.2, hence, due to statistical independence among E, T, F -0.4 ⇒ P(good) = P(E = 0)P(T = 0)P(F’) = e-0.1 e-0.3 (1 – 0.2) = e ×0.8 ≅ 0.536 Note: alternatively, we can let D be the combined number of earthquakes or/and tornadoes, with mean rate νD = νE +νT = 0.1 + 0.3 = 0.4 (disasters per year), and compute -0.4 P(D = 0)P(F’) = e ×0.8 instead (b) In each year, P(good year) ≡ p = 0.536 (from (a)). Hence P(2 out of 5 years are good) ⎛ 5⎞ = ⎜⎜ ⎟⎟ p 2 (1 – p) 3 ≅ 0.287 ⎝ 2⎠ (c) Let’s work with D as defined in (a). P(only one incidence of natural hazard) = P(D = 0)P(F) + P(D = 1)P(F’) –0.4 –0.4 =e ×0.2 + (e ×0.4)×(1 – 0.2) ≅ 0.349 3.19 Mean rate of accident = 1/3 per year (a) P(N=0 in 5 years) = e-1/3x5 = 0.189 (b) Mean rate of fatal accident = 1/3x0.05 = 0.0167 per year P(failure between inspection) = 1-P(no fatal accident within 3 years) = 1-e-0.0167x3 = 0.0489 3.20 (a) Let X be the number of accidents along the 20 miles on a given blizzard day. X has a 1 ×20 miles = 0.4, hence Poisson distribution with λX = 50 miles –0.4 P(X ≥ 1) = 1 – P(X = 0) = 1 – e = 1 – 0 670320046 ≅ 0.33 (b) Let Y be the number of accident-free days among five blizzard days. With n = 5, and p = daily accident-free probability = P(X = 0) ≅ 0 670, we obtain ⎛ 5⎞ P(Y = 2) = ⎜⎜ ⎟⎟ p 2 (1 – p) 3 ≅ 0.16 ⎝ 2⎠ 3.21 (a) Let X be the number of accidents in two months. X has a Poisson distribution with 3 ×2 months = 0.5, hence λX = 12 months –0.5 P(X = 1) = e × 0.5 ≅ 0.303, whereas –(3/12)(4) 2 [(3/12)(4)] / 2! P(2 accidents in 4 months) = e –1 = e /2! ≅ 0.184 No, P(1 accidents in 2 months) and P(2 accidents in 4 months) are not the same. (b) 20% of all accidents are fatal, so the mean rate of fatal accidents is νF = νx×0.2 = 0.05 per month Hence the number of fatalities in two months, F has a Poisson distribution with mean λF = (0.05 per month)(2 months) = 0.1, hence –0.1 P(fatalities in two months) = 1 – P(F = 0) = 1 – e ≅ 0.095 3.22 The return period τ (in years) is defined by τ = 1 where p is the probability of flooding per year. p Therefore, the design periods of A and B being τA = 5 and τB = 10 years mean that the respective yearly flood probabilities are P(A) = 1/ (5 years) = 0.2 (probability per year) P(B) = 1/(10 years) = 0.1 (probability per year) (a) P(town encounters any flooding in a given year) = P(A∪B) = P(A) + P(B) - P(AB) = P(A) + P(B) - P(A)P(B) (∵ A and B s.i.) = 1 τA + 1 τB − 1 (i) τ AτB = 0.2 + 0.1 - 0.2·0.1 = 0.28 (b) Let X be the total number of flooded years among the next half decade. X is a binomial random variable with parameters n = 5 and p = 0.28. Hence P(X ≥ 2) = 1 - P(X < 2) = 1 - f(0) - f(1) = 1 - (1 - 0.28)5 - 5×(0.28)×(1 - 0.28)4 ≈ 0.43 (c) Let τA and τB be the improved return periods for levees A and B, respectively. Using (i) from part (a), we construct the following table: New τA (and cost) New τB (and cost) 10 ($5M) 10 ($5M) 20 ($20M) 20 ($20M) 20 ($10M) 30 ($20M) 20 ($10M) 30 ($20M) Yearly flooding probability 1 1 1 = + − τ A τ B τ Aτ B 0.145 0.130 0.0975 0.0817 Total cost (in million dollars) 5 + 10 = 15 5 + 20 = 25 20 + 10 = 30 20 + 20 = 40 Since the goal is to reduce the yearly flooding probability to at most 0.15, all these options will work but the top one is least expensive. Hence the optimal course of action is to change the return periods of A and B to 10 and 20 years, respectively. 3.23 Given: ν = mean flaw rate = 1 (flaw) / 50m2; t = area of a panel = 3m x 5m = 15m2 Let X be the number of flaws found on area t. X is Poisson distributed, i.e. f(x) = e −λ λx x! with λ = νt = 15 / 50 = 0.3 (flaws) (a) P(replacement) = 1 - P(0 or 1 flaw) = 1 - f(0) - f(1) = 1 - e-0.3 - 0.3 e-0.3 ≈ 0.037 (b) Since the probability of replacement is 0.037 and there are 100 panels, we would expect 0.037×100 = 3.7 replacements on average, which gives the expected replacement cost 3.7×$5000 ≅ $18500 (c) For the higher-grade glass, ν = 1 (flaw) / 80m2 ⇒ λ = 15/80 = 0.1875, with which we can calculate the new probability P(replacement) = 1 - e-0.1875 - 0.1875 e-0.1875 ≈ 0.0155, which gives rise to an expected replacement cost of 100×0.0155×$5100 ≈ $7920 (assuming each higher grade panel is also $100 more expensive to replace) We can now compare the total costs: Let C = initial cost of old type panels Old type: Expected total cost = C + Expected replacement cost = C + $18500 New type: Expected total cost = C + Extra initial cost + Expected replacement cost = C + $100×100 + $7920 = C + $17920, which is less than that of the old type, ∴ the higher grade panels are recommended. 3.24 (a) Let X be the number of trucks arriving in a 5-minute period. Given: truck arrival has mean rate ν = 1 (truck) / minute; hence with t = 5 minutes ⇒ λ = νt = 5 (trucks) Hence P(X ≥ 2) = 1 - P(X < 2) = 1 - P(X = 0) - P(X = 1) = 1 - e-5 - 5e-5 ≈ 0.96 (b) Given: P(a truck overloads) ≡ p = 0.1 ; number of trucks ≡ n = 5. We can use the binomial distribution to compute P(at most 1 truck overloaded among 5) = b(0; n, p) + b(1; n, p) = (1 - 0.1)5 + 5·(1 - 0.1)4·(1 - 0.1) ≈ 0.92 (c) We will first find probability of the compliment event. During t = 30 minutes, the average number of trucks is λ = νt = 1×30 = 30 (trucks). Any number (x) of trucks could pass, with the (Poisson) probability f(x) = λx e-λ / x! but if none of them is to be overloaded (event NO), the (binomial) probability is P(NO | x trucks pass) = 0.9x Hence the total probability of having no overloaded trucks during 30 minutes is P(NO | x = 0)P(x = 0) + P(NO | x = 1)P(x = 1) + …… ∞ 30 x = ∑ 0.9 x e − 30 x! x =0 ∞ -30 =e 27 x ∑ x = 0 x! = e-30 e27 = e-3 , hence the desired probability –3 P(any overloaded trucks passing) = 1 – e ≅ 0.95 3.25 (a) Let X be the number of tornadoes next year. X has a Poisson distribution with 20 occurences = 1 (per year); ν= 20 years t = 1 year ⇒ λ = νt = 1, hence P(next year will be a tornado year) = P(X ≥ 1) = 1 – P(X = 0) -λ = 1 – e = 1 – 1/e ≅ 0.632 (b) This is a binomial problem, with n = 3 and p = yearly probability of having any tornado(es) = 0.6321. Hence P(two tornado years in three years) ⎛ 3⎞ ⎛2⎞ = ⎜ ⎟ 0.632 2 (1 − 0.632)1 ≅ 0.441 (c) (i) (ii) The tornadoes follow a Poisson model with mean rate of occurrence ν = 1 (per year), hence for an observation period of t = 10 (years), the expected number of tornadoes is λ = νt = 10 Number of tornado years follows a binomial model with n = 10, p = 0.632120559, hence the mean (i.e. expected) number of tornado years over a ten-year period is np ≅ 6.32 3.26 (a) Let X be the number of strong earthquakes occurring within the next 20 years. X has a Poisson distribution with λ = (1/50 years)(20 years) = 0.4, hence P(X ≤ 1) = P(X = 0) + P(X = 1) = e-0.4 (1 + 0.4) ≅ 0.938 (b) Let Y be the total number (maximum 3) of bridges that will collapse during a strong earthquake; Y has a binomial distribution with parameters n = 3, p = 0.3, hence the required probability is P(Y = 1) = 3×0.3×(1 – 0.3)2 = 0.441 (c) During a strong earthquake, the probability that all 3 bridges survive is (1 – 0.3)3 = 0.73 = 0.343. But since any number of strong earthquakes could happen during the next 20 years, we need to compute the total probability that there is no bridge collapse (with X as defined in part (a)), ∞ P(no bridge collapses) = ∑ P(no bridge collapses | x earthquakes occur in 20 years)P(X = x) x =0 ∞ = ∑ (0.343)x [(e-0.4)(0.4x) / x!] x =0 ∞ = (e-0.4) ∑ [(0.343)(0.4)]x / x! x =0 = (e-0.4)(e0.1372) = e-0.2628 ≅ 0.769 3.27 (a) Let X be the total number of excavations along the pipeline over the next year; X has a Poisson distribution with mean λ = (1/50 miles)(100 miles) = 2, hence P(at least two excavations) = 1 – P(X = 0) – P(X = 1) = 1 – e-λ (1 + λ) = 1 – e-2(3) ≅ 0.594 (b) For each excavation that takes place, the pipeline has 0.4 probability of getting damaged, and hence (1 – 0.4) = 0.6 probability of having no damage. Hence P(any damage to pipeline | X = 2) = 1 – P(no damage | X = 2) = 1 – 0.62 = 1 – 0.36 = 0.64 Alternative method: Let Di denote “damage to pipeline in i-th excavation”; the desired probability is P(D1∪ D2) = P(D1) + P(D2) – P(D1D2) = P(D1) + P(D2) – P(D1 | D2) P(D2) = P(D1) + P(D2) – P(D1) P(D2) = 0.4 + 0.4 – 0.42 = 0.8 – 0.16 = 0.64 (c) Any number (x) of excavations could take place, but there must be no damage no matter what x value, hence we have the total probability ∞ ∑ P(no damage | x excavations) P(x excavations) x =0 ∞ = ∑ e −λ λ x x! (0.6λ) x -λ 0.6λ =e e x! x 0.6 x =0 ∞ = e-λ ∑ x =0 -0.4λ -0.4(2) =e =e ≅ 0.449 -0.8 =e Alternative method: recall the meaning of ν in a Poisson process—it is the mean rate, i.e. the true proportion of occurrence over a large period of observation. Experimentally, it would be determined by ν= nE N where nE is the number of excavations observed over a very large number (N) of miles of pipeline. Since 40% of all excavations are damaging ones, damaging excavations must also occur as a Poisson process, but with the “diluted” mean rate of 0.4n E νD = = 0.4ν , hence N νD = (0.4)(1/50) = (1/125) (damaging excavations per mile) Hence P(no damaging excavation over a 100 mile pipeline) –(1/125 mi.)(100 mi.) = e–100/125 = e–0.8 =e ≅ 0.449 3.28 (a) Let X be the number of flaws along the 30-inch weld connection. X has a Poisson distribution with 0 .1 λX = ×30 inches = 0.25, hence 12 inches –0.25 P(acceptable connection) = P(X = 0) = e ≅ 0.779 (b) Let Y be the number of acceptable connections among three. Y has a binomial distribution with n = 3 and p = P(an acceptable connection) = 0.778800783, hence the desired probability is P(Y ≥ 2) = P(Y = 2) + P(Y = 3) = 3 p 2 (1 − p) + p 3 ≅ 0.875 (c) The number of flaws (F) among 90 inches of weld has a Poisson distribution with 0 .1 λF = ×90 inches = 0.75, hence 12 inches Hence P(1 flaw in 3 connections) = P(1 flaw in 90 inches of weld) = P(F = 1) –0.75 =e ×0.75 ≅ 0.354 Note: “1 flaw in 3 connections” is not the same as “1 unacceptable connections among 3”, as an unacceptable connection does not necessarily contain only 1 flaw—it could have 2, 3, 4, etc. 3.29 Mean rate of flood occurrence = 6/10 = 0.6 per year (a) P(1 ≤ N ≤ 3 within 3 years) (0.6 × 3) 1 (0.6 × 3) 2 (0.6 × 3) 3 − 0.6×3 + +]e =[1! 2! 3! = (1.8+1.62+0.972)x0.165 = 0.725 (b) Mean rate of flood that would cause inundation = 0.6x0.02 P(treatment plant will not be inundated within 5 years) = P(no occurrence of inundating flood) = e-0.02 = 0.98 3.30 Given: both I and N are Poisson processes, with νI = 0.01/mi and νN = 0.05/mi. Hence, along a 50-mile section of the highway, I and N have Poisson distributions with respective means λI = (0.01/mi)(50 mi) = 0.5, λN= (0.05/mi)(50 mi) = 2.5 (a) P(N = 2) = e − λ N (λN)2 / 2! = e–2.5 2.52 / 2 ≅ 0.257 (b) A, accident of either type, has the combined mean rate of occurrence νA = νI + νN = 0.06, hence A has a Poison distribution with λA = (0.06/mi)(50 mi) = 3, thus P(A ≥ 3) = 1 – P(A = 0) – P(A = 1) – P(A = 2) –3 2 = 1 – e (1 + 3 + 3 /2) ≅ 0.577 (c) Let us model this as an n = 2 binomial problem, where each “trial” corresponds to an accident, in which “success” means “injury” and “failure” means “non-injury”. How do we determine p? Consider the physical meaning of the ratio νI : νN = 0.01 : 0.05 -- it compares how frequently I occurs relative to N, hence their relative likelihood must be 1 to 5, thus P(N) = 5/6 and P(I) = 1/6 whenever an accident occurs. Therefore, p = 5/6, and 2 2 P(2 successes among 2 trials) = p = (1/6) ≅ 0.028 3.31 (a) Let W and S denote the number of winter and summer thunderstorms, respectively. Their mean occurrence rates (in numbers per month) are estimated based on the observed data as (i) νW ≅ (173)/(21×6) ≅ 1.37; (ii) νS ≅ (840)/(21×6) = 6.67 (b) Let us “superimpose” the two months as one, with winter and summer thunderstorms (X) that could happen simultaneously, at a combined mean rate of νX = (173 + 840)/(21×6) = 8.03968254 (storms per month), thus λX = (νX)(1 month) = 8.03968254, hence -8.040 4 P(X = 4|t = 1 month) = e (8.040)/4! ≅ 0.056 (c) In any particular year, the (Poisson) probability of having no thunderstorm in December is -173 / 126 = 0.253, P(W = 0 | t = 1 month) = e let’s call this p. Having such a “success” in 2 out of the next 5 years has the (binomial) probability ⎛ 5⎞ 2 ⎜⎜ ⎟⎟ p (1 − p)3 = 5!/2!/3!× 0.2532×(1 - 0.253)3 ≅ 0.267 ⎝ 2⎠ 3.32 (a) (b) (c) Let D denote defects and R denote defects that remain after inspection. Since only 20% of defects remain after inspection, we have 1 νR = νD×0.2 = ×0.2 = 0.001 per meter 200 meters For t = 3000 meters of seams, undetected defects have a mean of λ = νR t = (0.001/m)(3000m) = 3, ⇒ P(more than two defects) = 1 – P(two or less defects) = 1 – [P(D = 0) – P(D = 1) – P(D = 2)] –3 2 = 1 – [e (1 + 3 + 3 /2!)] = 1 – 0.423 ≅ 0.577 Suppose the allowable fraction of undetected defects is p (0 < p < 1), then the mean rate of undetected defects is νU = νD×p, hence, for 1000 meters of seams, the (Poisson) mean number of defects is νU×1000m = (1/200m)(p)(1000m) = 5p, thus if we require P(0 defects) = 0.95 –5p ⇒ e = 0.95 ⇒ –5p = ln(0.95) ⇒ p = – ln(0.95)/5 = 0.0103, i.e. only about 1% of defects can go on undetected. 3.33 Let J1 and J2 denote the events that John’s scheduled connection time is 1 and 2 hours, respectively, where P(J1) = 0.3 and P(J2) = 0.7. Also, let X be the delay time of the flight in hours. Note that P(X > x) = 1 – P(X ≤ x) = 1 – F(x) = 1 – (1 – e -x/0.5 =e -x/0.5) (a) Let M denote the event that John misses his connection, i.e. the flight delay time exceeded his scheduled time for connection. Using the theorem of total probability, P(M) = P(M | J1)P(J1) + P(M | J2)P(J2) = P(X > 1)×0.3 + P(X > 2)×0.7 = e– 1/0.5 × 0.3 + e– 2/0.5 × 0.7 = 0.135×0.3 + 0.0183×0.7 ≅ 0.053 (b) Regardless of whether John has a connection time of 1 hour and Mike has 2, or the opposite, for them to both miss their connections the flight must experience a delay of more than two hours, and the probability of such an event is P(X > 2) = e – 2/0.5 ≅ 0.018 (c) Since Mary has already waited for 30 minutes, the flight will have a delay time of at least 0.5 hours when it arrives. Hence the desired probability is P(X > 1 | X > 0.5) = P(X > 1 and X > 0.5) / P(X > 0.5) = P(X > 1) / P(X > 0.5) –1/0.5 –0.5/0.5 –2 –1 =e /e = e / e = 1/e ≅ 0.368 3.34 1000 rebars delivered with 2% below specification (a) 20 rebars are tested; hence, N = 1000, m = 20, n = 20 P(all 20 bars will pass test) = P(zero bars will fail test) ⎛ 20 ⎞⎛ 980 ⎞ ⎜ ⎟⎜ ⎟ ⎜ 0 ⎟⎜ 20 ⎠ = = 0.665 ⎛1000 ⎞ ⎜ ⎟ ⎜ 20 ⎟ ⎝ ⎠ (b) P(at least two bars will fail) = 1 – P(at most 1 bar will fail) = 1 – P(x=0) – P(x=1) ⎛ 20 ⎞⎛ 980 ⎞ ⎜ ⎟⎜ ⎟ ⎜ 1 ⎟⎜ 19 ⎟ ⎝ ⎠⎝ ⎠ = 1 – 0.665 – 1000 ⎛ ⎞ ⎜ ⎟ ⎜ 20 ⎟ ⎝ ⎠ = 1 – 0.665 – 0.277 = 0.058 (c) Let n be the required number of bars tested P(all n bars will pass test) ⎛ n ⎞⎛ 980 ⎞ ⎜ ⎟⎜ ⎟ ⎜ 0 ⎟⎜ n ⎟ ≅ 0.10 = ⎛1000 ⎞ ⎜ ⎟ ⎜ n ⎟ n ≅ 107.712 ⇒ 108 bars required 3.35 (a) “A gap larger than 15 seconds” means that there is sufficient time (15 seconds or more) between the arrival of two successive cars. Let T be the (random) time (in seconds) between successive cars, which is exponentially distributed with a mean of E(T) = (1/10) minute = 0.1 minute = 6 seconds, –15/6 –2.5 hence P(T > 15) = 1 – P(T ≤ 15) = e =e ≅ 0.082 (b) For any one gap, there is (1 – e–2.5) chance that it is not long enough for the driver to cross. –2.5 3 For such an event to happen 3 times consecutively, the probability is (1 – e) ; followed by –2.5 a long enough gap that allows crossing, which has probability e . Hence the desired probability is –2.5 3 –2.5 (1 – e) (e) ≅ 0.063 More formally, if N is the number of gaps one must wait until the first “success” (i.e. being able to cross), G follows a geometric distribution, n–1 –2.5 P(N = n) = (1 – p) p where p = e (c) Since the mean of a geometric distribution is 1/p (see Ang & Tang Table 5.1), he has to wait a mean number of –2.5 2.5 1/p = 1/ e =e ≅ 12.18 gaps before being able to cross. (d) First let us find the probability of the compliment event, i.e. P(none of the four gaps were large enough) –2.5 4 = (1 – e) Hence) = 1 – 0.70992075 ≅ 0.290 –2.5 4 P(cross within the first 4 gaps) = 1 – (1 – e 3.36 Standard deviation of crack length= 6.25 = 2.5 (a) Let X be the length (in micrometers) of any given crack.. ⎛ 74 − 71 ⎞ = 1 − Φ 1.2 = 1.2 P(X > 74) = 1 − Φ ⎜ ( ) ⎟ ⎝ 2.5 ⎠ (b) Given that 72 < X , the conditional probability P(X > 77 X > 72) = P(X > 77 and X > 72) / P(X > 72) = P(X > 77) / P(X > 72) = [1 - Φ((77 - 71)/2.5)] / [1 - Φ((72 - 71)/2.5)] = [1 - Φ(2.4)] / [1 - Φ(0.4)] =0.0082 / 0.8446 ≅ 0.024 3.37 (a) P(activity C will start on schedule) = P(A120) 120 − 100) = 1 − Φ (2) = 0.02275 = 1 − Φ(10 (b) P(CE-W) = P(A>115) 115 − 100) = 1 − Φ (1.5) = 0.0668 = 1 − Φ(10 (c) P(C) = P(CN-S)P(N-S)+P(CE-W)P(E-W) = 0.02275x0.8 + 0.0668x0.2 = 0.0315 3.40 Let X be the settlement of the proposed structure; X ~ N(μ X , σ X). Given probability: P(X ≤ 2) = 0.95 Also, since we’re given the c.o.v. = Hence, P(X ≤ 2) = Φ (or and σX = 0.2 , σ X = 0.2 μ x μX 2 − μx) = 0.95 0.2 μ x 2 − μx = Φ −1 (0.95) = 0.95 = 1.645 0.2 μ x μ x = 1.5 P(X > 2.5) = 1 − Φ (2.5 − 1.5) = 1 − Φ (3.063) = 0.00047 0.2 × 1.5 3.41 (a) Let X be her cylinder’s strength in kips. To be the second place winner, X must be above 70 but below 100, hence P(70 < X < 100) 100 − 80 70 − 80 = Φ( ) − Φ( ) 20 20 = Φ (1) − Φ (−0.5) = 0.841 − 0.309 = 0.532 (b) P(X > 100 | X > 90) = P(X > 100 and X > 90) / P(X > 90) = P(X > 100) / P(X > 90) = {1 − Φ[(100 − 80)/20]} / {1 − Φ[(90 − 80)/20]} = [1 − Φ(1)] / [1 − Φ(0.5)] = 0.159 / 0.309 ≅ 0.514 (c) Let Y be the boyfriend’s cylinder strength in kips, which has a mean of 1.01×80 = 80.8. Therefore, Y ~ N(80.8, σY) Let D = Y – X; D is normally distributed with a mean of μD = μY - μX = 80.8 − 80 = 0.8 > 0 This suffices to conclude that the guy’s cylinder is more likely to score higher. Mathematically, 0 − 0.8 P(D > 0) = Φ ( ) > 0.5 σD hence it is more likely for the guy’s cylinder strength to be higher than the girl’s. 3.42 H = annual maximum wave height = N(4,3.2) 6−4 (a) P(H>6) = 1 − Φ ((b) Design requirement is P(no exceedance over 3 years) = 0.8 = (1-p)3 Where p = p(no exceedance in a given year) 3.2) = 1 − Φ (0.625) = 0.266 Hence, p = 1-(0.8)1/3 = 0.0717 That is, Φ (h−4 where h is the designed wave height) = 0.0717 3.2 h = 3.2 Φ -1(0.0717) + 4 = 3.2x0.374 + 4 = 5.2 m (c) Probability of damaging wave height in a year = 0.266x0.4 = 0.1064 Hence mean rate of damaging wave height = 0.1064 per year P(no damage in 3 years) = P(no damaging wave in 3 years) = e-0.1064x3 =0.727 3.43 Let X be the daily SO2 concentration in ppm, where X ~ N(0.03, 0.4×0.03), i.e., X is normal with mean 0.03 and standard deviation 0.012. Thus the weekly mean, X ~ N(0.03, 0.012/ 7). Hence 0.04 − 0.03 ⎞ 1. P(X < 0.04) = Φ ⎛⎜ ⎟ = Φ(2.205) = 0.986 , hence ⎝ 0.012 / 7 ⎠ P1(violation) = 1 − 0.9863 = 0.0137, whereas 2. On any one day, the probability of X being under 0.075 is 0.075 − 0.03 ⎞ P(X < 0.075) = Φ ⎛⎜ ⎟ = Φ(3.75) ≅, hence 0.012 ⎠ ⎝ P(no violation) = P(0 or 1 day being under 0.075 out of 7 days) 7 6 = 1 + 7×1 ×(1 − 1) =1 = P2(violation) = 1 − 1 =0 < P1(violation), ∴criteria 1 is more likely to be violated, i.e. more strict. 3.44 Let F be the daily flow rate; we're given F ~ N(10,2) (a) P(excessive flow rate) = P(F > 14) ⎛ 14 − 10 ⎞ = 1− Φ⎜ ⎟ ⎝ 2 ⎠ = 1 − Φ (2) = 1 − 0.97725 ≈ 0.02275 (b) Let X be the total number of days with excessive flow rate during a three-day period. X follows a binomial distribution with n = 3 and p = 0.02275 (probability of excessive flow on any given day), hence P(no violation) = P(zero violations for 3 days) = P(X = 0) = (1 - p)3 ≅ 0.933 (c) Now, with n changed to 5, while p = 0.02275 remains the same, P(not charged) = P(X = 0 or X = 1) = P(X = 0) + P(X = 1) = (1 - p)5 + 5·p·(1 - p)4 ≅ 0.995 which is larger than the answer in (b). Since the non-violation probability is larger, this is a better option. (d) In this case we work backwards—fix the probability of violation, and determine the required parameter values. We want P(violation) = 0.01 ⇒ P(non-violation) = 0.99 ⇒ (1 - p)3 = 0.99 ⇒ p = 0.00334, but recall from part (a) that p is obtained by computing P(F > 14) = 0.003344 =1 − Φ ⎛ 14 − μ F ⎞ ⎜ ⎟ ⎝ σF ⎠ hence, 14 − μ F =Φ -1 (0.99666) = 2.712 ⇒ μ F ≅ 8.58 2 3.45 (a) The "parameters" λ and ζ of a log-normal R.V. are related to its mean and standard deviation μ and σ as follows: ⎛σ ⎞ ζ = ln(1 + ⎜ ⎟) ⎝ μ⎠ 2 2 λ = ln μ − Substituting the given values δT ≡ ζ2 2 σT = 0.4, μT = 80 , we find μT ζ 2 = 0.14842 and λ = 4.307817, hence ζ ≈ 0.385 and λ ≈ 4.308 The importance of these parameters is that they are the standard deviation and mean of the related variable X ≡ ln(T), while X is normal. Probability calculations concerning T can be done through X, as follows: ⎛ ln 20 − 4.3078 ⎞ = Φ (−3.406) = 0.00033 ⎝ 0.385 ⎠ P (T ≤ 20) = Φ⎜ ⎟ (b) (c) "T > 100" is the given event, while "a severe quake occurs over the next year" is the event "100 < T < 101", hence we seek the conditional probability P(100 < T < 101 | T > 100) = P(100 < T < 101 and T > 100) / P(T > 100) = P(100 < T < 101) / P(T > 100) = P(ln 100 < ln T < ln 101) / P(ln T > ln 100) ⎛ ln101 − λ ⎞ ⎛ ln100 − λ ⎞ Φ⎜ −Φ⎜ ⎟ ⎟ ⎝ ζ ⎠ ⎝ ζ ⎠ = ⎛ ln100 − λ ⎞ 1− Φ⎜ ⎟ ⎝ ζ ⎠ = 0.787468 − 0.779895 1 − 0.779895 = 0.034 3.46 (a) Let X be the daily average pollutant concentration. The parameters of X are: ζ ≅ δ = 0.2, λ ≅ ln 60 – 0.22 / 2 ≅ 4.074 Hence P(X > 100) = 1 – P(X ≤ 100) = 1 − Φ(ln100 − 4.074) 0.2 1 − Φ(2.654) = 1 – 0.996024 ≅ 0.004 (b) Let Y be the total number of days on which critical level is reached during a given week. Y follows a binomial distribution with parameters n = 7, p = 0.004 (and 1 – p = 0.9964), hence the required probability is P(Y = 0) = 0.9967 ≅ 0.972 3.47 ζ = ln(1 + δ 2) = 0.2936 H 1 2 λ = ln μ − ζ 2 = 2.953 (a) P(25 m long pile will not anchor satisfactorily) = P(H>25-2) = P(H>23) = 1 − Φ((b) P(H>2TH>24) = ln 23 − 2.953) = 1 − Φ (0.622) = 0.267 0.2936 P(H > 27 ∩ H > 24) P(H > 27) = P(H > 24) P(H > 24) ln 27 − 2.953) 1 − Φ (1.677) 0.0468 0.2936 = = = 0.21 = ln 24 − 2.953 − Φ 1 (0.767) 0.222 1 − Φ( ) 0.2936 1 − Φ(3.48 (a) Let X be the pile capacity in tons. X is log-normal with parameters ζX ≅ δX = 0.2, λX ≅ ln 100 – 0.22 / 2 ⎛ ln100 − 4.585 ⎞ = 1 − Φ 0.1 = 1 − 0.54 = 0.46 P(X > 100) = 1 - P(X ≤ 100) = 1 − Φ⎜ ( ) ⎟ ⎝ 0.2 ⎠ (b) Let L be the maximum load applied; L is log-normal with parameters δL = 15/50 = 0.3 2 1/2 ⇒ ζL = [ln(1 + 0.3)] = 0.294 ⇒ λL = ln 50 – 0.2942 / 2 = 3.869 It is convenient to formulate P(failure) as P(X < L) = P(X / L < 1) = P(ln(X/L) < ln 1) = P(ln X – ln L < 0) but D = ln X – ln L is the difference of two normals, so it is again normal, with 2 2 1/2 μD = λX – λL = 0.716 and σD = (ζX + ζL) = 0.355 ⎛ 0 − 0.716 ⎞ = Φ −2.017 = 0.0219 ⇒ P(D < 0) = Φ⎜ ( ) ⎟ ⎝ 0.355 ⎠ (c) P(X > 100 | X > 75) = P(X > 100 and X > 75) / P(X > 75) = P(X > 100) / P(X > 75) ln 75 − 4.585)] = [answer to (a)] / [1 − Φ(0.2 = 0.46 / [1 − Φ(−1.338)] = 0.46 / Φ(1.338) = 0.46 / 0.9096 ≅ 0.506 (d) P(X > 100 | X > 90) = P(X > 100 and X > 90) / P(X > 90) = P(X > 100) / P(X > 90) ln 90 − 4.585 = [answer to (a)] / [1 − Φ( )] 0.2 = 0.460172104 / [1 − Φ(− 0.427)] = 0.46 / Φ(0.427) = 0.46 / 0.665 ≅ 0.692 3.49 ⎛σ ⎞ ζ = ln(1+ ⎜ ⎟) ⎝ μ⎠ 2 2 λ = ln μ − ζ2 2 (a) Let X be the maximum wind velocity (in mph) at the given city. X ~ LN(λ, ζ) where ζ = ln(1+ δ 2) ≅ δ = 0.2, thus λ = ln μ − ζ2 ≅ ln (90) − 0.22 / 2 2 Since ζ and λ are the standard deviation and mean of the normal variate ln(X), ⎛ ln120 − 4.48 ⎞ = 1 − Φ(1.538) = 1 − 0.938≅ 0.062 P(X > 120) = 1 − Φ⎜ ⎟ ⎝ 0.2 ⎠ (b) To design for 100-year wind means the yearly probability of exceeding the design speed (V) is 1/100 = 0.01, i.e. P(X > V) = 0.01, i.e. P(X ≤ V) = 0.99 ⎛ ln V − 4.48 ⎞ = 0.99 Φ⎜ ⎟ 0.2 ⎝ 0.2 ⎠ ln V − 4.48 = Φ −1 (0.99) = 2.33 0.2 V = 141(mph) 3.50 T = time until breakdown = gamma with mean of 40 days and standard deviation of 10 days (a) k/ν = 40, c.o.v. = 0.25 = Hence, k = 16, ν = 0.4 1 k Let t be the required maintenance schedule interval Hence, P(T 58) = 1− (c) I (0.4 × 65,16) I (26,16) P(T > 65) 0.01417 = 1− u = 1− u = 1− = 0 .7 P(T > 58) I u (0.4 × 58,16) I u (23.2,16) 0.048 P(at least one out of three will breakdown) = 1 – P(none of the three will breakdown) = 1 – (0.95)3 = 0.143 3.51 The capacity C is gamma distributed with a mean of 2,500 tone and a c.o.v. of 35% k/ν = 2500, (a) (b) 1 = 0.35 k Hence, k = 8.16, ν = 0.00327 P(C >3000C > 1500) P (C > 3000) 1 − I u (0.00327 × 3000, 8.16) 1 − I u (9.81, 8.16) 0.745 = = = = 0.838 = P (C > 1500) 1 − I u (0.00327 × 1500, 8.16) I u (4.905, 8.16) 0.889 P(C5) = P(T>5T1 = 2)P(T1 = 2) + P(T>5T1 = 3)P(T1 = 3) = P(T2>3)P(T1 = 2) + P(T2>2)P(T1 = 2) = 0x1/4 + 1/4x3/4 = 0.1875 (b) P(T>5) = P(T>5N=0)P(N=0) + P(T>5N=1)P(N=1) + P(T>5N=2)P(N=2) + P(T>5N=3)P(N=3) Given 0 ship in the queue, P(T>5) = 0 Given 1 ship in the queue, P(T>5) = P(T1>5) = 0 Given 2 ships in the queue, P(T>5) = 0.1875 from above Given 3 ships in the queue, the waiting time will be at least 6 days. Hence it is certain that P(T>5) = 1 In summary, P(T>5) = 0x0.1 + 0x0.3 + 0.1875x0.4 + 1x0.2 = 0.275 3.53 Traffic volume= V = Beta between 600 and 1100 vph with mean of 750 vph and c.o.v. of 0.20 A = accidenct (a) P(Jamming occurs on the bridge) = P(J) = P(JA)P(A)+P(J A)P(A) = 1x0.02 + P(V>1000)x0.98 For the parameters of Beta distribution for V, q (1100-600) q+r qr (0.2x750)2 = (1100-600)2 2 (q + r) (q + r + 1) Hence, r ≈ 0.95,q ≈ 0.41 750 = 600+ P(V>1000) = 1- β u (0.41, 0.95) But u = (1000-600)/(1100-600) = 0.8 P(V>1000) = 1- β 0.8 (0.41, 0.95) = 0.097 P(J) = 0.02 + 0.097x0.98 = 0.971 3.54 By using the binomial distribution, P(2 out of 8 students will fail) ⎛8⎞ = ⎜⎜ ⎟⎟(0.2) (0.8) 2 2 6 ⎝ ⎠ = 28x(0.2)2(0.8)6 = 0.294 By using the hyper geometric distribution, we need number of passing students in a class of 30 = 24 number of failing students in a class of 30 = 6 Hence P(2 out of 8 students will fail) ⎛ 24 ⎞⎛ 6 ⎞ ⎜ ⎟⎜ ⎟ 28 × 15 ⎝ 6 ⎠⎝ 2 ⎠ = = 5852925 ⎛ 30 ⎞ ⎜ ⎟ ⎝8⎠ =0.00007 3.55 (a) T = trouble-free operational time follows a gamma distribution with mean of 35 days and c.o.v. of 0.25 k/ν = 35, 1 = 0.25 k Hence, k = 16, ν = 0.457 P(T>40) = 1-Iu(0.457x40, 16) = 1-Iu(18.3, 16) = 0.736 (b) For the total of 50 road graders, number of graders, Number of graders with T 5.5) P(C > 5.5) = P(C > 4) P (C > 4) ln 5.5 − ln 6.5 1 − Φ( ) 0.907 0 . 198 = = = 0.915 ln 4 − ln 6.5 0.9929 1 − Φ( ) 0.198 P(C>5.5C>4) = Mean rate of damaging earthquake = 0.2x1/500 = 0.0004 P(building survives 100 years) = P(no damaging earthquake in 100 years) = e-0.0004x100 = e-0.04 = 0.96 (d) P(survival of at least 4 structures during the earthquake) ⎛5⎞ ⎛ 5⎞ = ⎜⎜ ⎟⎟(0.8) (0.2) + ⎜⎜ ⎟⎟(0.8) (0.2) ⎝ 5⎠ ⎝ 4⎠ = 0.4096+0.3277 = 0.737 4 1 5 0 mean rate earthquake that causes damage to at most 3 structures = 0.737x1/500 = 0.00147 P(at least four building will survive 100 years) = e-0.00147x100 = e-0.147 = 0.863 3.57 (a) Let A and B denote the pressure at nodes A and B, respectively. Since A is log-normal with mean = 10 and c.o.v. = 0.2 (small), we have ζ A ≅ 0.2, and λA = ln(μA) – ζ A2 2 ≅ ln(10) – 0.02 = 2.282585093 ⇒ P(satisfactory performance at node A) ⎛ ln14 − 2.283 ⎞ − Φ ⎛ ln 6 − 2.283 ⎞ = Φ 1.788 − Φ −2.4560 = 0.955 = P (6 < A < 14) = Φ⎜ ( ) ( ) ⎜ ⎟ ⎟ 0.2 0.2 ⎝ ⎠ ⎝ ⎠ (b) Let Ni denote the event of pressure at node i being within normal range; i = A,B. Given: P(NB) = 0.9 ⇒ P(N B) = 0.1; P(N B | N A) = 2×0.1 = 0.2 Hence P(unsatisfactory water services to the city) = P(N A · N B) = P(N B | N A)P(N A) = 0.2×(1 – answer to (a)) = 0.2×(1 – 0.955319) ≅ 0.0089 (c) The options are: (I) to change the c.o.v. of A to 0.15: repeating similar calculations as done in (a), we get the new values of: λA ζA 2.2913 0.15 lower limit for Z –3.33 upper limit for Z 2.318 P(normal pressure at A) 0.98934 hence the probability of unsatisfactory water services, P(N B | N A)P(N A) becomes 0.2×(1 – 0.9893) ≅ 0.0021 (II) to change P(N B) to 0.05, and hence P(N B | N A) = 2×0.05 = 0.10, thus P(N B | N A)P(N A) = 0.10×(1 – 0.9556) ≅ 0.0044 ⇒ Option I is better since it offers a lower probability of unsatisfactory water services than II. 3.58 (a) fX(x) is obtained by “integrating out” the independence on y, 1 y3 ⎤ 6 6⎡ ∴fX(x) = (x + y 2)dy = ⎢ xy + ⎥ 5 5⎣ 3 ⎦0 0 2 (0 < x < 1) = (3x + 1) 5 1 ∫ (b) fY|X(y|x) = f X ,Y (x, y) = f X (x) (6 / 5)(x + y 2) x + y2 =3 (2 / 5)(3 x + 1) 3x + 1 1 ∫f Hence P(Y > 0.5 | X = 0.5) = Y |0.5 (y | x = 0.5)dy 0.5 1 ⎡ y3 ⎤ 0 .5 + y 2 dy = (3/2.5) ⎢0.5 y + ⎥ 1 .5 + 1 3 ⎦ 0.5 ⎣ 0.5 1 =3 ∫ = 0.65 1 1 (c) E(XY) = 6 xyf X ,Y (x, y) dxdy = 5 0 ∫∫ 0 1 = 1 1 ∫ ∫ (x 2 y + xy 3)dxdy 0 0 1 2 3 3 ydy + y dy = 1/5 + 3/20 = 7/20 = 0.35 50 50 ∫ ∫ = Cov(X,Y) = E(XY) − E(X)E(Y) = 0.35 − (3/5)(3/5) = -0.01, while σX = {E(X2) − [E(X)]2}1/2 = [(13/30) − (3/5)2]1/2 = 0.271 σY = {E(Y2) – [E(Y)]2}1/2 = [(11/25) – (3/5)2]1/2 = 0.283, Hence the correlation coefficient, ρXY = Cov(X , Y) σ XσY = −0.01 0.271 × 0.283 ≅ - 0.131 3.59 (a) Summing over the last row, 2nd & 3rd columns of the given joint PMF table, we obtain P(X ≥ 2 and Y > 20) = 0.1 + 0.1 = 0.2 (b) Given that X = 2, the new, reduced sample space corresponds to only the second column, where probabilities sum to (0.15 + 0.25 + 0.10) = 0.5, not one, so all those probabilities should now be divided by 0.5. Hence P(Y ≥ 20 | X = 2) = 0.25 / 0.5 + 0.1 / 0.5 = 0.35 / 0.5 = 0.7 (c) If X and Y are s.i., then (say) P(Y ≥ 20) should be the same as P(Y ≥ 20 | X = 2) = 0.7. However, P(Y ≥ 20) = 0.10 + 0.25 + 0.25 + 0.0 + 0.10 + 0.10 = 0.80 ≠ 0.7, hence X and Y are not s.i. (d) Summing over each row, we obtain the (unconditional) probabilities P(Y = 10) = 0.20, P(Y = 20) = 0.60, P(Y = 30) = 0.20, hence the marginal PMF of runoff Y is as follows: fY(y) 0.6 0.2 0.2 10 20 30 y (e) Given that X = 2, we use the probabilities in the X = 2 column, each multiplied by 2 so that their sum is unity. Hence we have P(Y = 10 | X = 2) = 0.15×2 = 0.30, P(Y = 20 | X = 2) = 0.25×2 = 0.50, P(Y = 30 | X = 2) = 0.10×2 = 0.20, and hence the PMF plot: fY|X(y|2) 0.5 0.3 0.2 10 20 30 y (f) By summing over each column, we obtain the marginal PMF of X as P(X = 1) = 0.15, P(X = 2) = 0.5, P(X = 3) = 0.35. With these, and results from part (d), we calculate E(X) = 0.15×1 + 0.5×2 + 0.35×3 = 2.2, 2 2 2 Var(X) = 0.15×(1 – 2.2) + 0.5×(2 – 2.2) + 0.35×(3 – 2.2) = 0.46, similarly E(Y) = 0.2×10 + 0.6×20 + 0.2×30 = 20, 2 2 2 Var(Y) = 0.2×(10 – 20) + 0.6×(20 – 20) + 0.2×(30 – 20) = 40, Also, E(XY) = ∑ x·y·f(x,y) all = 1×10×0.05 + 2×10×0.15 + 1×20×0.10 + 2×20×0.25 + 3×20×0.25 + 2×30×0.10 + 3×30×0.10 = 45.5 Hence the correlation coefficient is E (XY) − E (X) E (Y) ρ= Var (X) Var (Y) = 45.5 − 2.2 × 20 0.46 × 40 ≅ 0.35 4.1 Since it leads to an important result, let us first do it for the general case where X ~ N(λ, ζ), i.e. f X (x) = 1 1 ⎛ x −λ ⎞ ⎟ − ⎜ 2 ⎝ ζ ⎠ 2 e 2π ζ To obtain PDF fY(y) for Y = eX, one may apply Ang & Tang (4.6), fY(y) = fX(g-1) dg −1 dy where y = g(x) = ex; ⇒ x = g-1(y) = ln(y) dg −1 d 1 ⇒ = ln y = dy dy y 1 since y = eX > 0 y Hence, expressed as a function of y, the PDF fY(y) is = fX(g-1) 1 dg −1 = fX(ln(y)) dy y 1 ⎛ ln y − λ ⎞ − ⎜ ⎟ 2⎝ ζ ⎠ 2 e (non-negative y only) 2π yζ which, by comparison to Ang & Tang (3.29), is exactly what is called a log-normal distribution with parameters λ and ζ, i.e. if X ~ N(λ, ζ), and Y = eX, then Y ~ LN(λ, ζ). Hence, for the particular case where X ~ N(2, 0.4), Y is LN with parameters λ being 2 and ζ being 0.4. 4.2 To have a better physical feel in terms of probability (rather than probability density), let’s work with the CDF (which we can later differentiate to get the PDF) of Y: since Y cannot be negative, we know that P(Y < 0) = 0, hence when y < 0: FY(y) = 0 ⇒ fY(y) = [FY(y)]’ = 0 But when y ≥ 0, FY(y) = P(Y ≤ y) = P(1 mX 2 ≤ y) 2 2y 2y) ≤X≤ m m = P(− ⎛ 2y ⎞ ⎛ 2y ⎞ − FX ⎜ − ⎟ ⎜ m ⎟ ⎜ m ⎟ = FX ⎜ ⎛ 2y ⎞ ⎜ m ⎟ ⎝ −0 ⎠ = FX ⎜ ⎛ 2y ⎞ ⎜ m ⎟ ⎝ ⎠ = FX ⎜ Hence the PDF, fY(y) = d [FY(y)] dy ⎛ 2y ⎞ d 2y ⎟⎟ m ⎜ dy m ⎝ ⎠ 8y 2y 1 exp(−) = 2 3 ma ma π 2my = fX ⎜ = 4 a3 2y 2y exp(−) 3 πm ma 2 Hence the answer is ⎧4 ⎪ f Y (y) = ⎨ a 3 ⎪ ⎩ 0 2y 2y exp(−) 3 πm ma 2 y≥0 y0.55x400) P(S) 220 − 400 120 − 200 60 − 100 ⎞)] ⎞)][1 − Φ( )][1 − Φ (40 40 15 = Φ (2.67)x Φ (2)x Φ (4.5) = [1 − Φ (= 0.99962x0.977x1 = 0.973 P(at least one source not able to supply respective allocation) = 0.027 4.4 (a) Let TJ be John’s travel time in (hours); TJ = T3 + T4 with μ T = 5 + 4 = 9 (hours), and σ T = [32 + 12 + (2)(0.8)(3)(1)]1/2 = 3.847 (hours) J J Hence P(TJ > 10 hours) = 1 − P(T J − μ TJ σT J ≤ 10 − 9) 3.847 = 1 − Φ(0.26) = 1 − 0.603 ≅ 0.397 (b) Let TB be Bob’s travel time in (hours); TB = T1 + T2 with μT = 6 + 4 = 10 (hours), and B σ T = [22 + 12]1/2 = 5 (hours) B Hence P(TJ – TB > 1) = P(TB – TJ + 1 < 0), now let R ≡ TB – TJ + 1; R is normal with μR = μTB – μTJ + 1 = 10 – 9 + 1 = 2, σR = [σ TB 2 + σ TJ 2]1/2 = (5 + 14.8)1/2 = 19.8 , hence P(R < 0) = Φ (0−2) 19.8 = Φ(-0.449) ≅ 0.327 (c) Since the lower route (A-C-D) has a smaller expected travel time of μ TJ = 9 hours as compared to the upper (with expected travel time = μTB = 10 hours), one should take the lower route to minimize expected travel time from A to D. 4.5 (a) To calculate probability, we first need to have the PDF of S. As a linear combination of three normal variables, S itself is normal, with parameters μS = 0.3×5 + 0.2×8 + 0.1×7 = 3.8 (cm) σS = 0.3 212 + 0.2 2 2 2 + 0.1212 ≅ 0.51 (cm) Hence 4 − 3 .8) 0.51 = 1 − Φ(0.3922) = 1 − 0.6526 ≅ 0.347 P(S > 4cm) = 1 -Φ ((b) Now that we have a constraint A + B + C = 20m, these variables are no longer all independent, for example, we have C = 20 – A – B Hence S = 0.3 A + 0.2 B + 0.1(20 – A – B) ⇒ S = 0.2 A + 0.1 B + 2, with ρAB = 0.5. Thus μS = 0.2×5 + 0.1×8 + 2 = 3.8 (cm) as before, and 0.2 212 + 0.12 2 2 + 2 × 0.5 × 0.1 × 1 × 2 = 0.12 ≅ 0.346 (cm) σS = Hence P(S > 4cm) = 1 − Φ(4 − 3.8) 0.12 = 1 − Φ(0.577) ≅ 0.282 4.6 Q = 4A + B + 2C = 4A + B +2(30 – A – B) = 2A –B + 60, hence μQ = 2×5 - 8 + 60 = 62, 2 2 1/2 σQ = [4×3 + 1×2 + 2(2)(-1)(-0.5)(3)(2)] = (36 + 4 + 12)1/2 = 52 Q − μ Q 40 − 62) Hence P(Q < 40) = P(< σQ 52 = Φ(-3.0508) ≅ 0.00114 4.7 (a) Let Q1 and Q2 be the annual maximum flood peak in rivers 1 and 2, respectively. We have Q1 ~ N(35, 10), Q2 ~ N(25, 10) The annual max. peak discharge passing through the city, Q, is the sum of them, Q = Q1 + Q2, hence μ Q = μ Q1 + μ Q2 = 35 + 25 = 60 (m3/sec), and σ Q2 = σ Q21 + σ Q2 2 + 2 ρ Q1Q2 σ Q1 σ Q2 2 2 = 10 + 10 + 2×0.5×10×10 = 300 (m3/sec), ⇒ σ Q = 300 ≅ 17.32 (m3/sec) (b) The annual risk of flooding, p = P(Q > 100) = 1 − Φ(100 − 60) 300 = 1 − Φ(2.309) = 1 − 0.9895 = 0.0105 (probability each year) Hence the return period is τ = = 1 p 1 = 95.59643882 0.0105 ≅ 95 years. (c) Since the yearly risk of flooding is p = 0.0105, and we have a course of n = 10 years, we adopt a binomial model for X, the total number of flood years over a 10-year period. P(city experiences (any) flooding) = 1 – P(city experiences no flooding at all) = 1 – P(X = 0) = 1 – (1 – p)10 = 1 – (1 – 0.0105)10 = 1 – 0.989510 = 10% (d) The requirement on p is, using the flooding probability expression from part (c): 1 – (1 – p)10 = 0.1 ÷ 2 = 0.05 ⇒ p = 1 – (1 – 0.05)1/10 = 0.0051, which translates into a condition on the design channel capacity Q0, following what’s done in (b), Q − 60) = 0.0051 1 − Φ(0 300 Q − 60) = 0.9949 ⇒ Φ(0 300 ⇒ Q0 = 60 + Φ-1(0.9949) = 60 + 2.57 300 = 104.5 300 ∴Extending the channel capacity to about 104.5 m3/sec will cut the risk by half. 4.8 P(T20)+P(Y>20)-P(X>20)P(Y>20) = (1 − Φ ((b) 20 − 20 20 − 15 20 − 20 20 − 15)) + (1 − Φ ( )) − (1 − Φ ( )) × (1 − Φ ( )) 4 3 4 3 =0.5+0.0478-0.5 × 0.0478 =0.524 Z in normal distribution with μ Z = 0.6 × 20 + 0.4 × 15 = 18 σ Z = 0.6 2 4 2 + 0.4 2 3 2 = 2.683 20 − 18) = 1 − 0.7718 = 0.2282 2.683 μ Z = 0.6 × 20 + 0.4 × 15 = 18 ∴ P(Z > 20) = 1 − Φ ((c) σ Z = 0.6 2 4 2 + 0.4 2 3 2 + 2 × 0.8 × 0.6 × 0.4 × 4 × 3 = 3.436 ∴ P(Z > 20) = 1 − Φ (20 − 18) = 1 − 0.72 = 0.28 3.436 4.13 (a) (b) (c) P(X≥2)=1-P(X=0)-P(X=1) ⎛5⎞ ⎛5⎞ = 1 − ⎜⎜ ⎟⎟0.6 0 0.4 5 − ⎜⎜ ⎟⎟0.610.4 4 ⎝0⎠ ⎝1⎠ = 1 − 0.4 5 − 5 × 0.6 × 0.4 4 = 0.046 NH, NB are the number of highway and building jobs won P(NH =1∩ NB =0)=P(NH =1)P(NB =0) ⎛ 3⎞ ⎛ 2⎞ = ⎜⎜ ⎟⎟0.610.4 2 ⎜⎜ ⎟⎟0.6 0 0.4 2 ⎝1⎠ ⎝0⎠ =0.046 T=H1+H2+B where H1, H2 and B are the profits from the respective jobs. T in Normal distribution with μ T = 100 + 100 + 80 = 280 σ T = 40 2 + 40 2 + 20 2 = 60 300 − 280) = 1 − Φ (0.333) = 0.2695 60 μ T = 100 + 100 + 80 = 280 ∴ P(T > 300) = 1 − Φ ((d) σ T = 40 2 + 40 2 + 20 2 + 2 × 0.8 × 40 × 40 = 78.5 ∴ P(T > 300) = 1 − Φ (300 − 280) = 1 − Φ (0.255) = 0.4 78.5 4.14 S: the amount of water available. S follows LogNormal Distribution with E(S)=1, c.o.v=0.4 ζ S2 = ln(1 + 0.4 2) = 0.14842 λ S = ln μ S − 12 ζ S2 = ln 1 − 0.5 × 0.14842 = −0.07421 D: the total demand of water. E(D)=1.5, c.o.v=0.1 ζ D2 = 0.12 = 0.01 λ D = ln μ D − 12 ζ D2 = ln 1.5 − 0.5 × 0.01 = 0.4005 P(water shortage)=P(D>S) =P(D/S>1) =P((Z=ln(D/S))>0) Z follows Normal Distribution with: Var (Z) = ζ 2 D + ζ S2 = 0.12 + 0.148 = 0.158 μ Z = λ D − λ S = 0.4005 − (−0.07421) = 0.4747 0 − 0.4747 ⇒ P (Z > 0)

= 1 − Φ ( ) = 0.883 0.158 4.15 (a) P is lognormally distributed with λP = λC + λR + 2λV – ln2 = (ln1.8 – 0.5x0.22) + (ln2.3x10-3 – 0.5x0.12) + 2[ln120-0.5xln(1+0.452)] – ln2 = 0.568 + (-6.08) + 2(4.6953) – 0.693 = 3.186 ξP ξ + ξ + 4ξ = 0.2 + 0.1 + 4 × ln(1 + 0.45) = 0.2 + 0.1 + 4 × 0.1844 2 C 2 R 2 V 2 2 2 2 = 0.887 ln 30 − 3.186) = 1 − Φ (0.243)
= 0.596 0.887 (b) P(P>30) = 1 − Φ((c) C is lognormally distributed with λC = ln90 – 0.5x0.152 = 4.4886 ξC = 0.15 P(Failure of antenna) = P(C 0) S − µ S 0 − (−100) >) = P(σS 15550 = 1 – Φ(0.80192694) ≅ 0.211 Let T denote the total (dead + vehicle) weight, T = V + D. T ~ N(1100 + 1200, 0.5 15550), hence P(V + D > 2500) = P(T > 2500) T − µ
T 2500 − 2300 > = P( ) σT 15550 = 1 − Φ(1.60385388) ≅ 0.054 4.24 (a) Let B be the total weight of one batch. µB = 40×2.5 kg = 100 kg (b) Since n > 30, we may apply the Central Limit Theorem: B is approximately normal with µB = 100 kg and σB = 40 × 0.1 kg, hence 101 − 100) 40 × 0.1) = 1 -Φ (1.581) = 1 – 0.9431 ≅ 5.69% 101 − 100) (c) In
this case, P(penalty) = P(B > 101) = 1 -Φ (40 × 1) = 1 -Φ (0.158) = 1 – 0.562816481 ≅ 43.7% P(penalty) = P(B > 101) = 1-Φ (This penalty probability is much higher, caused by the large standard deviation which makes the PDF occupy more area in the tails. Hence a large standard deviation (i.e. uncertainty) is undesirable. 4.25 Over the 50-year
life, the expected number of change of occupancy is 25. (a) The PDF of the lifetime maximum live load Yn or Y20 in this case is, from Eq. 3.57, f Yn (y) = f Y20 (y) = 20[Fx (y)]19 f x (y) However X is lognormal with parameters λ, ξ whose values are ξ= ln(1 + 0.3 2) = 0.294 λ = ln12 – 0.5x0.2942 = 2.44 1 ln y − 2.44 2)] exp[− (0.294 2 2π (0.294)
y ln y − 2.44) FX (y) = Φ(0.294 ln y − 2.44 19 1 1 ln y − 2.44 2] exp[− (f Y20 (y) = 20[Φ)] 0.294 0.737 y 2 0.294 (b) 1 f X (y) = Hence By following the results of Examples 3.34 and 3.37, Y20(y) will converge asymptotically to the Type I distribution with parameters un = 2.94(2 ln 20 − and α n = ln ln 20 + ln 4π)+2.44 = 2.187 2 2 ln 20 2 ln 20 =
8.326 0.294 νn = e2.187 = 8.9 k = 8.326 f Y20 (y) = 8.326 8.9 9.326 8.9 ( ) exp[−( ) 8.326] y y 8.9 4.26 (a) f X (x) = ν (νx) k −1 e −νx Γ(k) Then, x FX (x) = ∫ ν (νz) k −1 e −νz Γ(k) 0 dz The CDF of the largest value from a sample of size n is (Eq. 4.2): FYn (y) = [FX (y)] = [∫ n ν (νz) k −1 e −νz Γ(k) 0 dz]n whereas, the PDF is (Eq. 4.3): n −1
fYn (y) = n[FX (y)] x f x (y) = n[∫ ν (νz) k −1 e −νz Γ(k) 0 (b) n −1 dz] ν (νx) k −1 e −νx Γ(k) If the distribution of the large value is Type I asymptotic, then according Eq. 4.77, d 1 lim dx [h (x)] = 0 x →∞ n The hazard function hn(x) is (Eq. 4.28): hn (x) = f X (x) = 1 − FX (x) ν (νx) k −1 e −νx / Γ(k) x 1 − ∫ν (νz) k −1 e −νz dz / Γ(k) 0 ν x k =
k −1 x e Γ(k) − ∫ ν z k 0 −νx k −1 e x k −1 e −νx = −νz dz x ν Γ(k) − ∫ z k −1 e −νz dz −k 0 x d lim dx [h x →∞ ν − k Γ(k) − ∫ z k −1e −νz e dz −ν 1 d] = lim [x → ∞ dx n (x) 0 k −1 −νx x e] Using Leibnitz rule for the derivative of the integral, the above limit becomes, x − x k −1e −νx x k −1e −νx − [ν − k Γ(k) − ∫ z k −1e −νz dz]e −νx [(k − 1) x k −
2 − νx k −1 0 k −1 −νx 2 lim (x e x →∞) x = −1− [ν − k Γ(k) − ∫ z k −1e −νz dz](k − 1 − νx) 0 lim x→∞ x k e −νx Recall that ∞ ∫z k −1 −νz e dz = 0 Γ(k) νk x −k k −1 −νz lim{[ν Γ(k) − ∫ z e dz](k − 1 − νx)} = 0 x →∞ 0 and k −νx lim (x e)=0 x →∞ Therefore, using the L’Hospital’s rule: x − 1 − lim − x k −1e −νx (k − 1 − νx) − ν [ν − k Γ(k) − ∫ z
k −1e −νz dz] x→∞ e −νx x k −1 (k − νx) − ν [ν −k 0 x Γ(k) − ∫ z k −1e −νz dz] − (k − 1 − νx) 0 − lim −νx k −1 k − νx e x (k − νx) x →∞ x →∞ ∞ The second term is of the form . Again, using L’Hospital’s rule: ∞ − (k − 1 − νx) ν = = −1 lim k − νx −ν x →∞ 0 The third term is of the form , thus, 0 νx k −1 e −νx =0 lim −νx [−νx k −1 (k − νx) + (k
− 1) x k − 2 (k − νx) − νx k −1] x →∞ e = −1− lim since the order of the polynomial in the denominator is higher than that of the numerator. Therefore, the distribution of the largest value converges to the Type I distribution. 4.27 (a) Let Z = X-18; Z follows an exponential distribution with mean = 3.2, i.e. λ = 1/3.2 Following the result of Example
3.33, the largest value of an exponentially distributed random variable will approach the Type I distribution. Also, for large n, the CDF of the largest value FYn (y) = exp(−ne − λy) which can be compared with the Type I distribution FYn (y) = exp(−e −α n (y −un)) By setting ne-λy = e −α n (y − u n) We obtain αn = λ and =e un = −α n y α n u n
e ln n λ For a 1 year period, the number of axle loads is 1355. Hence mean maximum axle load is u Y1355 = 18 + [u n + σY 1355 = ln 1355 0.577 γ = +] = 42.9 α n 1 / 3.2 1 / 3.2 π 2 π 2 π 2 (3.2) 2 = = 16.8 6 6α n2 ∴δ Y1355 = 16.8 = 0.39 42.9 Similarly for period of 5, 10 and 25 years, the number of axle loads are 6775, 13550 and 33875 respectively.
The corresponding mean values and c.o.v. are as follows: µY6775 = 48 σY6775 = 16.8 δY6775 = 0.35 (b) µY13550 = 50.3 σY13550 = 16.8 δY13550 = 0.34 µY33875 = 53.2 σY33875 = 16.8 and δY33875 = 0.316 For a 20 year period, n = 27100 FYn (y) = exp(−e −α n (y −18−un)) = exp[−e − (1/ 3.2)(y −18−(ln 27100)3.2)] Hence the probability
that it will subject to an axle load of over 80 tone is 1 − FY27100 (80) = 1 − exp[−e −(1/ 3.2)(80−18−(ln 27100) 3.2)] = 1 – 0.999896 = 0.000104 (c) For an exceedance probability of 10%, the “design axle load” L can be obtained from 1 − exp[−e − (1/ 3.2)(L −18−(ln 27100) 3.2)] ≡ 0.1 Hence, exp[−e − (1/ 3.2)(L −50.7)] = 0.9 L = 57.9 tons
4.28 (a) Daily DO level = N(3, 0.5) From E 4.18, the largest value for an initial variate of N(µ , σ) follows a Type I Extreme Value distribution. Because of the symmetry between the left tails of the normal distribution, it can be shown that the monthly minimum DO level also follows a Type I smallest value distribution with u1 = − 2 ln 30 + ln ln 30 + ln
4π 2 2 ln 30 α 1 = 2 ln 30 / 0.5 = 5.21 + 3 = 1.112 Similarly, for the annual minimum DO, it also follows the Type I smallest value distribution with u1 = − 2 ln 365 + ln ln 365 + ln 4π + 3 = −3.435 + 1.227 + 3 = 0.792 2 2 ln 365 α 1 = 2 ln 365 / 0.5 = 6.87 (b) P[(Y1)30 < 0.5] = 1 – exp[-e5.21(0.5-1.112)] = 0.04 P[(Y1)365 < 0.5] = 1 – exp[-e6.87(0.5-
0.792)] = 0.125 (c) For Type I smallest value distribution µ 1 = u1 − 0.577 α1 ; π2 σ1 = 2 6α 1 Hence for monthly minimum DO level, 0.577 = 1.0mg / l ; µ 1 = 1.112 − 5.21 Similarly for the annual minimum level, 0.577 = 0.708mg / l ; µ 1 = 0.792 − 6.87 σ1 = π2 6 × 5.212 σ1 = = 0.06mg / l π2 6 × 6.87 2 = 0.035mg / l 4.29 Let Y be the maximum
wind velocity during a hurricane Y follows a Type I asymptotic distribution with µY = 100 and σY = 0.45 From Equations 3.61a and 3.61b, the distribution parameters un and αn can be determined as follows: π2 = (0.45 × 100) 2 or 2 6α n π αn = = 0.0285 6 (0.45 ×100) 0.5772 u n = 100 − = 79.75 0.0285 (a) P(Damage during a hurricane) = P(Y>150)
= 1 − FY (150) = 1 − exp[−e −0.0285(150 − 79.75)] = 0.126 (b) Let D be the revised design Hence, 1 − FY (D) = 1 − exp[−e −0.0285(D − 79.75)] = 0.063 D = 175.6 kph (c) Mean rate of damaging hurricane = ν = (1/200) × 0.126 = 0.00063 P(Damage to original structure over 100 years) = 1 – P(no damaging hurricane in 100 years) = 1 –
exp(-0.00063 × 100) = 0.061 P(damage to revised structure over 100 years) = 1 - e-(0.00063/2)(100) = 0.031 (d) P(at least one out of 3 structure with original design will be damaged over 100 years) = 1 – P(none of the 3 structure damaged) = 1 - (1-0.061)3 = 0.172 4.30 (a) Daily maximum wind velocity is N(40, 10) The maximum over 3 months will
converge to a Type I largest value distribution whose CDF is (from Example 4.18 FYn (y) = exp[−e −α n (y − u n)] with un = 2 ln n − ln ln n + ln 4π +µ 2 2 ln n α n = 2 ln n / σ For a 3 month period, use n = 91 Hence un = 2 ln 91 − ln ln 91 + ln 4π + 40 = 42.1 2 2 ln 91 α n = 2 ln 91 / 10 = 0.3 (b) The most probable wind velocity during the 3-
month period is 42.1 mph P(Damage) = P(Yn > 70) = 1 − exp[−e −0.3(70 − 42.1)] = 0.00023 (c) Mean of Yn = un + γ/αn = 42.1 + 0.577/0.3 = 44 mph σ of Yn = π /(6α n) = π /(0.3 6) = 4.277 and c.o.v. of Yn = 4.277/44 = 0.097 4.31 (a) Daily maximum wind velocity is lognormal with mean 40 and c.o.v. of 25%. The maximum wind velocity over a 3-
month period will converge to a Type II, or the Fisher-Tippett distribution, whose CDF is νn FYn (y) = exp[−(y)k] with the most probable value νn and shape parameters k ν n = eu and k = αn ln ln n + ln 4π un = ζ (2 ln n −)+λ 2 2 ln n αn = and n 2 ln n ζ using the results obtained in Example 4.21 In this case, n = 91 ; ζ = 0.25 1 2 λ = ln 40 −
(0.25) 2 = 3.658 Hence the most probable wind velocity during erection period is ν n = exp[0.25(2 ln 91 − = exp[4.18] = 65.4 mph Moreover, k = 2 ln 91 / 0.25 = 12 (b) P(Damage) = P(Yn > 70) = 1 − exp[−(= 0.36 65.4 12)] 70 ln ln 91 + ln 4π) + 3.658 2 2 ln 91 4.32 h= LV 2 f 2 Dg Variable L D f V (a) Mean Value 100 ft 1 ft 0.02 9.0 fps c.o.v. 0.05
0.15 0.25 0.2 std. Deviation 5 ft 0.15 ft 0.005 1.8 fps Assuming the above random variables are statistically independent, the first order mean and variance of the hydraulic head loss h are, respectively, µh ≈ 100(9) 2 (0.02) = 2.516 ft 2(1)(32.2) µL µ 2 2 µL µ 2 2 µ 2µ µ 2 µV2 µ f 2 2 2) +σD(2) +σ f ( ) + σ V2 (L) σ ≈σ (µ D (2 g) µ (2 g) µ (2 g) µ (2 g
) 2 h V 2 L V D D V f D = 100 × 2 × 9 × 0.02 2 100 × 92 2 100 × 92 2 92 × 0.02 2)) + 1 .8 2 ( ) + 0.0052 ( ) + 0.152 (2 52 (1(2 × 32.2) 1(2 × 32.2) 1(2 × 32.2) 1(2 × 32.2) = 0.0158+355.9433+0.3955+1.01246 = 357.367 Hence, σ h ≈ 18.9 ft (b) The second order mean of the hydraulic head loss 1 2 µ h = 2.516 + σ V2 (µL µV µ f 1 (2))) + σ D2 (3
µ D (2 g) 2 µ D (2 g) 2µ L µ f 2 2 1 2 2 × 100 × 0.02 1 2 100 × 9 × 0.02) + 0.15 (3 (2)) = 2.516 + 1.8 (2 1(2 × 32.2) 2 1 (2 × 32.2) = 2.516 + 0.1006 + 0.056 = 2.673 ft 4.33 D= PL3 , 3EI Variable P E b h I= bh 3 12 ⇒D= Mean Value 500 lb 3,000,000 psi 72 inch 144 inch 4 PL3 Ebh 3 c.o.v. 0.2 0.25 0.05 0.05 std. Deviation 100 lb 750,000 psi 3.6 inch
7.2 inch L = 15 ft = 180 inch (a) Assuming the above random variables are statistically independent, the first order mean and variance of the deflection D are, respectively, 4 × 500 × 1803 = 1.8 × 10 −5 inch 3 3000000 × 72 × 144 L3 2 4 µ P L3 2 4 µ P L3 × 3 2 4 µ P L3 2 2 2 2 2 2) +σE(2) + σb( ) +σh ( ) σD ≈ σP(3 3 2 3 4 µD ≈ µ E µb µ h + 2 ×
ρ bh × σ b × σ h × = 100 2 (µ µb µ h E 4µ P L3 × 3 µ E µ b2 µ h µ E µb µ h µ E µb µ h 4µ P L3 × 3 µ E µ b µ h4 4 × 500 × 180 3 2 4 × 500 × 180 3 2 4 × 180 3 2 2 2)) 3 . 6 ( ) 750000 (+ + 3e6 × 72 2 × 144 3 (3e6) 2 × 72 × 144 3 3e6 × 72 × 144 3 + 7.2 2 (4 × 500 × 180 3 × 3 2 4 × 500 ×180 3 × 3 4 × 500 × 180 3 × 3) + 2 × 0 . 8 × 3 . 6 × 7 . 2
× × 3e6 × 72 ×144 4 3e6 × 72 2 ×144 3 3e6 × 72 ×144 4 = 1.308x10-11 + 2.044x10-11 + 8.176x10-13 + 7.359x10-12 + 3.925x10-12 = 4.562x10-11 Hence, σ D ≈ 6.75x10-6 ft (b) The second order mean of the deflection 3 4 µ P L3 (−3)(−4) 4 µ P L3 (−1)(−2) 2 1 2 4 µ P L (−1)(−2) 2 2 µ D = 1.8 × 10 + [σ E ( ) +σb ( ) +σ h ( )] 2 µ 3E µ b µ h3 µ E µ
b3 µ h3 µ E µ b µ h5 −5 = 1.8 × 10 −5 + 750000 2 2 3 .6 2 × 2 7.2 2 × 12 4 × 500 × 180 3 + 3 × 1] [(3e6) 3 × 72 × 144 3 3e6 × 72 × 144 3 3e6 × 72 × 144 5 2 = 1.94x10-5 ft 4.34 (a) Using Ang & Tang Table 5.1, E(X) = (4 + 2) / 2 = 3, Var(X) = (4 - 2)2 / 12 = 1/3; E(Y) = 0 + (3 - 0) / (1 + 2) = 1, Var(Y) = 1×2×(3 - 0)2/(1+2)2(1+2+1) = 1/2; Also, for
W, since we know its median is 1, i.e. 1 ∫ λe λ dx = 0.5 ⇒ 1 − e λ = 0.5 ⇒ λ = ln 2 , hence − x − 0 E(W) = 1/λ ≈ 1.443, Var(W) = 1/λ2 ≈ 2.081 (b) Since Z = XY2W0.5, we compute the approximate values E(Z) ≈ E(X) [E(Y)]2 [E(W)]0.5 = 3×12×1.4430.5 ≈ 3.6, and ⎡ ∂Z ⎤ ⎡ ∂Z ⎤ ⎡ ∂Z ⎤ Var(Z) ≈ ⎢ Var (X) + ⎢ ⎢Var (Y) + ⎢ Var (W) ⎢ ⎣ ∂X ⎦ µ ⎣ ∂Y ⎦ µ ⎣ ∂W
⎦ µ 2 [2] 2 [2] 2 [ ] 2 = Y 2W 0.5 µ Var (X) + 2 XYW 0.5 µ Var (Y) + 0.5 XY 2W −0.5 µ Var (W) where µ denotes "evaluated at the mean values X = 3, Y = 1, W = 1.443", hence Var(Z) ≈ 29.7, 29.7 ≈ 1.51 Thus the c.o.v. of Z is 3.6 4.35 (a) Let X’ ≡ ln(X). Since X is log-normal, X’ is normal with parameters σX’ ≅ δX = 0.20, and µX’ ≅ ln 100 – δX2 / 2
= ln 100 – 0.202 / 2 = 4.585170186 When L is a constant (0.95), we have Y = (1 – 0.95)X = 0.05 X Taking the log of both sides and letting Y’ ≡ ln(Y) ⇒ Y’ = ln 0.05 + X’, i.e. Y’ is a constant plus a normal variate, hence Y’ is normal with parameters µY’ = ln 0.05 + µX’ ≅1.589437912, and σY’ = σX’ ≅ 0.20 Hence P(acceptable) = P(Y ≤ 8) = P(Y’ ≤ ln 8) Y
’− µ Y ' ln 8 − 1.589437912 = P( ) ≤ σY' 0.20 = Φ(2.450018146) ≅ 0.993 (b) (i) When L is also a random variable, Y = (1 – L)X is a non-linear function of two random variables, hence we need to estimate its mean and variance by E(Y) ≅ Y(L = µL, X = µX) = (1 – µL)(µX) = (1 – 0.96)(100) = 4, and ⎛ ∂Y ⎞ ⎛ ∂Y ⎞ Var(Y) ≅ ⎢ ⎢ Var (L) + ⎢ ⎢ Var (X) ⎝ ∂L ⎠ µ ⎝
∂X ⎠ µ 2 2 = (− X)2X =100,L =0.96 (0.012) + (1 − L)2X =100,L =0.96 (100 × 0.2) 2 = 10000×0.0001 + 0.0016×400 = 1.64 (ii) Assuming a log-normal distribution for Y and using its estimated mean and variance from (i), we have the approximate parameter values δY ≅ 1.64 , hence 4 ξY = ln(1 + δ Y2) ≅ ln(1 + 1.64 / 16) = 0.312; 2 λY ≅ ln 4 – 0.312
/ 2 = 1.338, thus ln 8 − 1.338 P(Y ≤ 8) = Φ ( ) 0.312 = Φ(2.375) ≅ 0.991 No, this offer from the contractor does not yield a larger performance acceptance probability. 4.36 (a) E(Qc)=0.463E(n)-1E(D)2.67E(S)0.5=0.463*0.015-1*3.02.67*0.0050.5=41.0ft3/sec Var(Qc)= [= ∂Qc ∂n ⎦µ ⎡ ∂Q ⎤⎡ ∂Q ⎤ Var (n) + ⎢ c ⎢ Var (D) + ⎢ c ⎢Var (S) ⎣ ∂D ⎦ µ ⎣ ∂S ⎦ µ 2
2 (− 0.463D + (0.463n 2.67 −1 S 0.5 D −2) 2 µ 2 ( ) 2 Var (n) + 0.463n −1 × 2.67 D 1.67 S 0.5 µ Var (D)) 2 D 2.67 × 0.5S −0.5 µ Var (S) =22.665(ft3/sec)2 (b) The percentage of contribution of each random variable to the total uncertainty: ∂Qc ∂n ⎡ ⎦µ 2 Var (n) / Var (Qc) = 74.2% ⎡ ∂Qc ⎤⎢ ∂D ⎢Var (D) / Var (Qc) = 21.2% ⎣ ⎦µ 2 ⎡ ∂Qc ⎤⎢ ∂S ⎢Var (
S) / Var (Qc) = 4.6% ⎣ ⎦µ 2 (c) QC follows the log-normal distribution with: µ Q = 41.0 C σ 2 QC = 22.665 σ 22.665 = = 0.116 µ 41 λ = ln µ − 12 ζ 2 = 3.707 ⇒ζ ≈ ∴ P(QC > 30) = 1 − Φ (ln 30 − 3.707) = 0.9958 0.116 4.37 50 (a) T= ∑ Ri i =1 µ T = 50 × 1 = 50 σ T = 50 × 12 = 7.07 ∴ P(T > 60) = 1 − Φ ((b) i. 60 − 50) = 1 − Φ (1.414) = 1 − 0.9213
= 0.078 7.07 V = N R ln(R + 1) =1*1*ln(1+1) =ln2 =0.693 ⎛ ∂V ⎞ ⎛ ∂V ⎞ Var(V)= ⎢ ⎢ Var (R) ⎢Var (N) + ⎢ ⎝ ∂R ⎠ ⎝ ∂N ⎠ 2 [ii. 2] 2 R ⎡ ⎤ = R ln(R + 1) (0.5) + ⎢ N (+ ln(R + 1)) ⎢ × 1 ⎣ R +1 ⎦ 2 2 2 =(ln2) (0.5) +(1/2+ln2) ×1 =0.48*0.25+1.424*1 =0.12+1.424 =1.544 1.544 = 1.793 ∴δ V = 0.693 The approximation is not expected to be good because:
2 2 a) R and N have large variance; b) The function is highly nonlinear Check the second order term for the mean: 1 ∂ 2V 1 ⎡ (R +1) − R 1 ⎤ · Var (R) = ⎢ N ( ) ⎢Var (R) = 0.375 + 2 2 2 ∂R 2 R 2⎣ R +1 ⎦ (R + 1) which is not that small compared to the first order term. ∴the approximations are not good. 4.38 w= (a) K M σK = 200 µK = 400, M = 100, The
first order mean and variance of w are 1 1 µk = 400 = 2 10 10 1 1 1 1 σ w2 ≈ σ K2 (µ K − 2) 2 = 200 2 ( ) 2 = 0.25 10 2 2 400 µw ≈ Hence, (b) σw = 0.5 M is also random with µM = 100 and σM = 20 The first order mean and variance of w are µw ≈ µK 400 = =2 100 µM 1 2 1 2 σ w2 ≈ 0.25 + σ M2 (µ K µ M −3 / 2) 2 = 0.25 + 20 2 ((20)(100) −3 / 2
( )) 2 = 0.25 + 0.04 = 0.29 Hence, σw = 0.538 (c) The second order approximate mean of w is 1 2 ∂2w 1 2 ∂2w µw ≈ 2 + σ K (2)µ + σ M ( )µ 2 2 ∂K ∂M 2 1 1 1 1 1 3 −3 / 2 µ M −1 / 2 µ M −1 / 2 + σ M2 (−)(−) µ K 1 / 2 µ M − 5 / 2 = 2 + σ K2 (−)( ) µ K 2 2 2 2 2 2 1 3 = − (200) 2 (400) − 3 / 2 (100) −1 / 2 + (20) 2 (400)1 / 2 (100) − 5 / 2 8 8 = 2 – 0.0625 +
0.03 = 1.968 5.1 MATHCAD statements: D := rnorm(100000 , 4.2 , 0.3) µL := 6.5 δL := σL := 0.8 λL := ln(µL) − 0.5 ·ζL 2 σL µL ζL := λL = 1.864 L := rlnorm(100000, λL , ζL) µw := 3.4 α := σw := 0.7 π α = 1.832 6 ·σw β := µw − 0.57721566 α β = 3.085 u := runif(100000 , 0 , 1) 1 W := β − α ⎛ (1 ⎞⎞ ⎝ ⎝ u ⎠⎠ ·ln⎜ ln⎜ → S := (D + L + W) n := 100 j
:= 0 .. n intj := 0 + 20 · j n h := hist(int, S) 1 .10 4 h 5000 0 0 5 10 int mean(S) = 14.102 µR := 1.5·mean(S) µR = 21.152 15 20 ( ) 2 ln 1 + δL δR := 0.15 ζR := ( ) 2 ln 1 + δR λR := ln(µR) − 1 2 ζR 2 ζR = 0.149 2 λR = 3.041 ·ζR R := rlnorm(100000 , λR , ζR) → x := [(R − S) < 0] 99999 ∑ pF := xi i= 1 100000 −3 pF = 9.76 × 10 The result obtained with
Mathcad with a sample size of 100,000 yields the probability of R 35000 30 j n therefore, b = 40 9999 ∑ p1 := s1i i= 1 10000 therefore, the probability that the annual cost of operating the waste treatment plant will exceed $35,000 is p1 = 0.3224 5.3 (a) Define water supply as random variable S, total demand of water is D MATHCAD statements: µS
:= 1 δS := 0.40 σS := µS ·δS (ζS := ln 1 + δS λS := ln(µS) − 1 2) 2 ζS ζS = 0.385 2 λS = −0.074 S := rlnorm(100000 , λS , ζS) µD := 1.5 δD := 0.1 ζD := δD λD := ln(µD) − D := rlnorm(100000 , λD , ζD) → x := (S < D) 1 2 2 ζD λD = 0.4 99999 ∑ pF := xi i= 1 pF = 0.884 100000 Define random variable Y=S/D, therefore event of water shortage is
Y 0.0) 99999 ∑ pf := xi i= 1 pf = 0.037 100000 (b) MATHCAD statements: → x := [(T2 + T3) < 60] 99999 ∑ pf := xi i= 1 pf = 0.55 100000 (c) MATHCAD statements: ζT2 := 0.2 µT2 := 20·2 λT2 := ln(µT2) − TRA := T2 + T3 1 2 2 ζT2 λT2 = 3.669 → xA := [(TRA) < 60] 99999 ∑ pfA := xAi i= 1 100000 pfA = 0.55 pfB := plnorm(60 , λT3 , ζT3) pfB =
0.933 2 1 pfA· + pfB· = 0.806 3 3 Therefore, 80.7% of passengers will arrive the shopping center during rush hours in less than one hour. (d) The passenger left Town B at 2:00pm, therefore the traffic is normal P(T345)=? MATHCAD statements: ζT3 := 0.3 µT3 := 40 λT3 := ln(µT3) − 1 2 ζT3 T3 := rlnorm(100000, λT3 , ζT3) → x := (T3 > 45) → y
:= (T3 > 45 ∧ T3 < 60) 99999 pf := ∑ yi ∑ xi i= 1 99999 i= 1 pf = 0.776 Therefore, the probability he will arrive on time is 0.776 Verify the simulation result by directly calculate the probability by cumulative distribution function pf1 := plnorm(60 , λT3 , ζT3) − plnorm(45 , λT3 , ζT3) pf2 := 1 − plnorm(45 , λT3 , ζT3) pf3 := pf1 pf2 pf3 = 0.773 5.5
S1 and S2 follow bivariate normal distribution (a) MATHCAD statements are: u1 := runif(100000 , 0 , 1) u2 := runif(100000 , 0 , 1) S1 := qnorm(u1 , 0 , 1) ·0.3 ·2 + 2 2 ⎡ ⎢ S2 := qnorm(u2 , 0 , 1) ·(0.3 ·2) · 1 − 0.7 + ⎢ 2 + 0.7 · → D := S1 − S2 n := 40 j := 0 .. n intj := 0 + 3 · j n h := hist(int, D) 1 .10 4 5000 h 0 0 1 2 3 int −4 mean(D) = 3.77 × 10
var(D) = 0.217 (b) the probability that D is less than 0.5 inch is → x := (D < 0.5) 99999 ∑ pF := xi i= 1 pF = 0.858 100000 (c) S1 and S2 are jointly lognormally distributed µS1 := 2 δS1 := 0.30 ζS1 := ( ) 2 ln 1 + δS1 ζS1 = 0.294 0.3 ·2 0.3 ·2 ⎤ ⎦ ·(S1 − 2) ⎦ λS1 := ln(µS1) − 1 2 ζS1 2 λS2 := λS1 λS1 = 0.65 ζS2 := ζS1 T1 := rlnorm(100000, λS1 ,
ζS1) A := log(S1) ⎡ ⎣ uB := ⎢λS2 + 0.7 · ζS2 ζS1 ⎤ ⎦ ·(A − λS1) ⎦ 2 sB := ζS2 · 1 − 0.7 B := qnorm(u2 , 0 , 1) ·sB + uB S2 := exp(B) → D := S1 − S2 n := 40 j := 0 .. n intj := 0 + 3 · h := hist(int, D) 2 .10 4 h 1 .10 4 0 0 1 2 3 int mean(D) = 0.481 var(D) = 0.321 The probability that D is less than 0.5 inch is: → x := (D < 0.5) 99999 ∑ pF := xi i= 1
100000 pF = 0.555 j n 5.6 (a) MATHCAD statements: µF1 := 35 σF1 := 10 ⎛ ln⎢ 1 + ζF1 := ⎢ ⎝ 2 σF1 ⎞ζF1 = 0.28 2 µF1 ⎠ λF1 := ln(µF1) − 1 2 2 λF1 = 3.516 ζF1 µF2 := 25 σF2 := 10 ⎛ ln⎢ 1 + ζF2 := ⎢ ⎝ 2 σF2 ⎞ζF2 = 0.385 2 µF2 ⎠ λF2 := ln(µF2) − 1 2 2 λF2 = 3.145 ζF2 F1 := rlnorm(100000, λF1 , ζF1) F2 := rlnorm(100000, λF2 , ζF2) F := F1 +
F2 n := 50 j := 0 .. n intj := 0 + 150 · j n 1 .10 4 h 5000 0 0 50 100 int mean(F) = 59.96 stdev(F) = 14.14 150 h := hist(int, F) (b) the annual risk that the city will experience flooding is the probability of flooding MATHCAD statements: → x := (F > C) C := 100 99999 ∑ PF := RP := xi i= 1 PF = 0.011 100000 1 RP = 92.678 PF The annual risk is
0.011, the corresponding return period is 92.7 years. (c) If the desired risk is reduced to half of the current one, i.e., 0.0055, then the capacity should be increased. C := for c ∈ 100 , 101 .. 300 → x ← (F > c) 99999 ∑ p← xi i= 1 100000 c if p > 0.0055 break if p ≤ 0.0055 c C←c C C = 107 Verify the above result: C := 107 → x := (F > C) 99999 ∑ PF :=
xi i= 1 100000 −3 PF = 5.02 × 10 Therefore, the channel capacity should be increased to 107m3/s. (d). Correlated case (d.a) MATHCAD statements: µF1 := 35 σF1 := 10 2 ⎛ σF1 ⎞⎢ ζF1 := ln 1 + 2 ⎢ µF1 ⎠⎞ ζF1 = 0.28 1 2 λF1 := ln(µF1) − ζF1 2 λF1 = 3.516 F1 := rlnorm(100000, λF1 , ζF1) A1 := ln(F1) µF2 := 25 σF2 := 10 2 ⎛ σF2 ⎞⎢ ζF2 := ln 1
+ 2 ⎢ µF2 ⎠⎞ ⎝ 1 2 λF2 := ln(µF2) − ζF2 2 ζF2 = 0.385 λF2 = 3.145 ρ := 0.80 uB1 := λF2 + ρ · ζF2 ζF1 sB1 := ζF2 · 1 − ρ ·(A1 − λF1) 2 u1 := runif(100000 , 0.0 , 1.0) B1 := qnorm(u1 , 0.0 , 1.0) ·sB1 + uB1 F2 := exp(B1) F := F1 + F2 n := 50 j := 0 .. n intj := 0 + 150 · j n h := hist(int, F) 1 .10 4 h 5000 0 0 50 100 150 int mean(F) = 60.019 stdev(F)
= 18.901 (d.b) the annual risk that the city will experience flooding is the probability of flooding MATHCAD statements: → x := (F > C) C := 100 99999 ∑ PF := RP := xi i= 1 100000 1 PF PF = 0.035 RP = 28.425 The annual risk is 0.035, the corresponding return period is 28.8 years. Compare with the independent case, it shows the annual risk of
flooding is increased due to the correlation between the flow rates of the two rivers. (d.c) If the desired risk is reduced to half of the current one, i.e., 0.0175, then the capacity should be increased. C := for c ∈ 100 , 101 .. 300 → x ← (F > c) 99999 ∑ p← xi i= 1 100000 c if p > 0.0175 break if p ≤ 0.0175 c C←c C C = 110 Verify the above result: C :=
110 → x := (F > C) 99999 ∑ PF := xi i= 1 100000 PF = 0.017 Therefore, the channel capacity should be increased to 110m3/s. 5.7 W := 200 W and g are contant values g := 32.2 Coefficient of friction, k, is beta distributed q := 3.0 r := 3.0 As q=r, then the distribution is symmetric. The median value is equal to the mean value. µk := 0.40 σk := 0.08
q+ r+ 1 q ·r ak := µk − q ·σk · ak = 0.188 q+ r+ 1 ·(q + r) q ·r bk := ak + σk · bk = 0.612 x := rbeta(100000 , 3.0 , 3.0) k := (bk − ak) ·x + ak n := 40 intj := 0 + 1.0 · j := 0 .. n j n h := hist(int, k) 1.5 .10 4 1 .10 4 h 5000 0 0 0.5 1 int Maximum ground acceleration, a, is a Type I extremal variate π α := α = 37.934 0.35 ·% ·0.3 ·g· 6 β := 0.3 ·g −
0.577216 α β = 9.645 u := runif(100000 , 0 , 1) a := β − n := 40 ⎛ (1 ⎞⎞ α ⎝ ⎝ u ⎠⎠ 1 ln⎜ ln⎜ j := 0 .. n intj := 0.0 + 15 · j n h := hist(int, a) 1 .10 5 h 5 .10 4 0 0 5 10 15 int (a) The resistance by friction R R := k ·W F := W · a g n := 40 intj := 0.0 + 100 · j := 0 .. n j n h := hist(int, F) 1 .10 5 h 5 .10 4 0 0 50 100 int → y := (R < F) 99999 ∑ pf := yi i= 1
100000 −1 pf = 1.181 × 10 The probability that a tank will slide from its base support is 0.119 (b) The resistance of the anchors for each tank is RA RA := for ra ∈ 0 , 0.1 .. 10 R ← k ·W + ra a g → x ← (R < F) F ← W· 99999 ∑ xi i= 1 pf ← 100000 ra if pf > (0.119 ·0.3) break if pf ≤ (0.119 ·0.3) ra RA ← ra RA = 8.2 RA Verify the above result: R := k ·W
+ RA a g → x := (R < F) F := W · 99999 ∑ pf := xi i= 1 100000 −2 pf = 3.512 × 10 (c) Considering epistemic uncertainties: assume the error of µk is mk, the error of µa is ma q := 3.0 r := 3.0 g := 32.2 µk := 0.40 σk := 0.08 Pf := mk ← rnorm(1000 , 1.0 , 0.25) ma ← rnorm(1000 , 1.0 , 0.45) for i ∈ 0 .. 999 ak ← µk ·mki − q ·σk · q+ r+ 1 q ·r q+ r+ 1 ·(
q + r) q ·r bk ← ak + σk · x ← rbeta(1000 , 3.0 , 3.0) k ← (bk − ak) ·x + ak α← π (0.35 ·% ·0.3 ·g) ·mai· 6 β ← 0.3 ·g·mai − 0.577216 α u ← runif(1000 , 0.0 , 1.0) a←β − 1 α ⎛ (1 ⎞⎞ ⎝ ⎝ u ⎠⎠ ln⎜ ln⎜ R ← k ·W a g → y ← (R < F) F ← W· 999 ∑ Pf i ← j= 1 1000 continue Pf yj n := 40 j := 0 .. n intj := 0.0 + 2 · j n hpf := hist(int, Pf) 600 400 hpf 200 0 0 1 2 int
mean(Pf) = 0.295 stdev(Pf) = 0.346 skew(Pf) = 0.876 v := sort(Pf) Pf90 := v900 v900 = 0.897 The 90% value for a conservative probability of sliding is 0.897 5.8 Single pile capacity T1, T2 µT1 := 20 δT1 := 0.20 (ζT1 :=) 2 ln 1 + δT1 λT1 := ln(µT1) − 1 2 λT2 := λT1 ζT1 = 0.198 2 ζT1 λT1 = 2.976 ζT2 := ζT1 T1 := rlnorm(100000, λT1 , ζT1) x1
:= log(T1) ⎡ ⎣ ux2 := ⎢λT2 + 0.8 · ζT2 ζT1 ⎤ ⎦ ·(x1 − λT1) ⎦ 2 sx2 := ζT2· 1 − 0.8 u2 := runif(100000 , 0 , 1) x2 := qnorm(u2 , 0 , 1) ·sx2 + ux2 T2 := exp(x2) T := T1 + T2 mean(T) = 25.151 stdev(T) = 4.392 covT := stdev(T) mean(T) n := 40 covT = 0.175 j := 0 .. n intj := 0 + 50 · j n 1.5 .10 4 1 .10 4 h 5000 0 0 20 40 int The external load is L µL :=
10 60 h := hist(int, T) δL := 0.30 ζL := ( ) 2 ln 1 + δL λL := ln(µL) − 1 2 ζL ζL = 0.294 2 ζL λL = 2.259 L := rlnorm(100000, λL , ζL) → y := (T < L) 99999 ∑ pf := yi −3 pf = 2.6 × 10 i= 1 100000 Therefore, the probability of failure of this pile group is 0.0026 Problem 5.9 Determine the solution to Problem 3.57 by MCS (a) f(x , y) := ( ) 6 2 x+ y 5 1 ⎡ 2
6 fX(x) := ⎢ f(x , y) dy → ·x + ⌡0 5 5 (b) 2 x+ y f(x , y) simplify→ 3 · fYx(x , y) := 3 ·x + 1 fX(x) 2 ⎡ 3 ·x + y FYx(x , y) := ⎢ fYx(x , y) dy simplify→ y· ⎢ 3 ·x + 1 ⎤ 1 ⎡ ⎢ 3 1 ⎤⎡ ⎢ ⎢2 ⎢ 1 1 ⎢ 2 ·⎣10 ·u2 + 2 ·(2 + 25 ·u2) ⎦⎦ − ⎢ 1 2 ⎢ 3 1 ⎤⎡ ⎢⎢ 2 ⎢10 ·u2 + 2 ·(2 + 25 ·u22) ⎢⎣ ⎦⎦ ⎢ 1 1 ⎡ ⎢ ⎢ 3 3 ⎢ ⎡ 1 1 ⎢ ⎡ ⎢ ⎢⎤ ⎢ ⎢ 2 2 1 1 2 ⎢1 2 2 simplify ⎢ −1 ⎢ ·⎣10 ·u2 + 2
·(2 + 25 ·u2) ⎦⎦ + + ·i·3 · ⎢ ·⎢⎣10 ·u2 + 2 ·(2 + 25 ·u2) ⎦⎦ + FYx(0.5 , y) − u2 → 1 2 2 solve, y ⎢ 4 ⎢ 3 ⎢ 1⎤ ⎡⎢ ⎢ ⎢ 2 ⎢ 2) ⎢ ⎢ ⎢ (2 ·⎣10 ·u2 + 2 · 2 + 25 ·u2 ⎦⎦ ⎢ 1 1 ⎡ ⎢ 3 3⎢ 1⎤ 1⎤ 1 ⎢ ⎡ ⎢ ⎡ ⎢ ⎢2 2 1 1 2 ⎢1 2 ⎢ −1 ·⎣10 ·u2 + 2 ·(2 + 25 ·u22) ⎦ + − ·i·3 · ·⎣10 ·u2 + 2 ·(2 + 25 ·u2) ⎦⎦ + ⎦⎢4 ⎣ 1 2 2 ⎦ ⎢ 3 1⎤⎡ ⎢ ⎢ ⎢ 2 ⎢ 2 ⎢ ⎢ 2 ·⎣10 ·u2 + 2 ·(2 + 25 ·u2) ⎦⎣
⎣ u2 := runif(100000 , 0 , 1) y should be real value and greater than zero, so ⎤ ⎢ ⎢ ⎢ ⎢ ⎢ 1 1 ⎢ 3 ⎢ 1 ⎢ 2 ⎢10 ·u2 + 2 ·(2 + 25 ·u22) ⎢ ⎢ ⎣ ⎦ ⎦ 2 3 1 ⎡⎢ ⎢ 2 2 ⎢ 1 ⎣10 ·u2 + 2 ·(2 + 25 ·u2) ⎦⎦ − 2 IFYx := · 2 1⎤⎡ ⎢ 2⎢10 ·u2 + 2 ·(2 + 25 ·u22) ⎢⎣ ⎦⎦ 1 3 → n := (
1.0 > IFYx ∧ IFYx > 0.5) 99999 ∑ p := ni i= 1 100000 p = 0.65 Verify the simulation result, 1.0 ⎛ ⎢ fYx(0.5 , y) dy → .65000000000000000000 ⌡0.5 (c) ⎡ 3 2 2 FX(x) := ⎢ fX(x) dx → ·x + ·x ⎣ 5 5 ⎦ 1⎤ ⎡ ⎢ −1 1 2 ⎢ + ·(1 + 15 ·u1) solve, x ⎢ 3 3 ⎢ FX (x) − u1 → ⎢ simplify 1 ⎢ ⎢ −1 1 2 ⎢ 3 − 3 ·(1 + 15 ·u1) ⎢ ⎣ ⎦x should be greater than zero, therefore the
inverse function of FX is u1 := runif(100000, 0 , 1) −1 1 2 IFX := + ·(1 + 15 ·u1) 3 3 1 ⎡ 3 6 2 fY(y) := ⎢ f(x , y) dx → + ·y ⌡0 5 5 ⎡ 2 3 3 FY(y) := ⎢ fY(y) dy → ·y + ·y ⎣ 5 5 ⌡ 2 ⎡ ⎤ ⎢ 3 1⎤⎡ ⎢ ⎢ ⎢ 2 ⎢ 2 1 ⎢ 2 ⎢ 1 1 ⎢10 ·u3 + 2 ·(2 + 25 ·u3) ⎦ − 2 ⎢ · ⎢ 1 2 ⎢ 3 1⎤ ⎡ ⎢ ⎢ ⎢ 2 ⎢ ⎢10 ·u3 + 2 ·(2 + 25 ·u32) ⎢⎣ ⎦⎦ ⎢ 2 2 ⎢ 3 3 1⎤ 1⎤ ⎡ ⎢ 1 ⎢ ⎢ 2 2 ⎢
solve, y ⎢ 1 −⎢⎣10 ·u3 + 2 ·(2 + 25 ·u32) ⎦⎦ + 2 + i·3 2 ·⎢⎣10 ·u3 + 2 ·(2 + 25 ·u32) ⎦⎦ + 2 ·i·3 2 ⎢ · FY (y) − u3 → 1 ⎢simplify ⎢ 4 ⎢ 3 1⎤ ⎡⎢ ⎢ ⎢ 2 ⎢ ⎢10 ·u3 + 2 ·(2 + 25 ·u32) ⎢⎣ ⎦⎦ ⎢ 2 2 ⎢ 3 3 1⎤ 1⎤ ⎡ ⎢ ⎢ ⎢ 1 1 ⎢ ⎢ 2 2 ⎢ ⎢ 2 2 2 2 ⎢ ⎢ ⎢ −1 · ⎣10 ·u3 + 2 ·(2 + 25 ·u3) ⎦ − 2 + i·3 ·⎣10 ·u3 + 2 ·(2 + 25 ·u3) ⎦ + 2 ·i·3 ⎢1 ⎢4 ⎢ ⎢ 3 1⎤ ⎡⎢ ⎢ ⎢2 ⎢
⎢10 ·u3 + 2 ·(2 + 25 ·u32) ⎢⎣ ⎦⎦ ⎢ ⎦ ⎣ Y should be greater than zero, therefore the inverse function of FY is u3 := runif(100000, 0 , 1) 2 3 1 ⎡⎢ ⎢2 2) ⎢⎢ (1 ⎣10 ·u3 + 2 · 2 + 25 ·u3 ⎦ − 2 IFY := · 2 1⎤⎡ ⎢ ⎢ 2 ⎢10 ·u3 + 2 ·(2 + 25 ·u32) ⎢⎣ ⎦⎦ 1 3 corr(IFX, IFYx) = −0.003 Yes, there are statistical correlation between X and Y. Check stdev(IFYx) = 0.2828
stdev(IFY) = 0.2817 ρ := ⎛ stdev(IFYx) ⎞ ⎝ stdev(IFY) ⎠ 1−⎢ 2 ρ = 0.09i 5.10 MathCAD statements u1 := runif(100000, 0.0, 1.0) X1 := for i ∈ 1 , 2 .. 99999 X1 ← 1 if u1 ≤ 0.15 i i X1 ← 3 if u1 > 0.65 i i X1 ← 2 otherwise i X1 u2 := runif(100000, 0.0, 1.0) Y1 := for i ∈ 1 , 2 .. 99999 Y1 ← 10 if u2 ≤ 1 Y1 ← 20 if u2 > 1 i i i i 3 3 ∧ X1 1 ∧ u2 ≤ 2 i i Y1 ← 30
if u2 ≥ 1 ∧ X1 i i 0.15 Y1 ← 20 if u2 ≤ 0.4 Y1 ← 30 if u2 ≥ 0.4 Y1 ← 10 if u2 ≤ 0 i i i i i i i i 0.15 0.5 ∧ X1 2 i 0.35 Y1 ← 30 if u2 ≥ 0.25 i Y1 → n := (X1 ≥ 2 ∧ Y1 > 20) 99999 PF := ∧ u2 > 0.25 i i ∑ 2 i Y1 ← 20 if u2 ≤ i (a) ∧ X1 0.5 0.5 n i i=1 100000 PF = 0.2 i 0.35 0.35 1 1 i Y1 ← 10 if u2 ≤ ∧ X1 3 i ∧ X1 i ∧ u2 > i ∧ X1 i 0.5 ∧ X1 2 i 3 0 0.35 3 ∧ X1 i 3 (b)
→ n := (X1 2) 99999 ∑ n i i=1 PF1 := PF1 = 0.5 100000 → m := (X1 2 ∧ Y1 ≥ 20) 99999 ∑ PF2 := PF3 := m i i=1 PF2 = 0.352 100000 PF2 PF3 = 0.704 PF1 The probability of the runoff Y is not less than 20 cfs when the precipitation is 2 inches is 0.7 (c) → n := (Y1 ≥ 20) 99999 ∑ PF4 := n i i=1 PF4 = 0.752 100000 as shown in (b) P(Y>=20|X=2)=0.7
P(Y>=20)=0.75 Therefore, X and Y are not independent (d) → n := (Y1 10) 99999 ∑ n i i=1 Py1 := Py1 = 0.197 100000 → n := (Y1 20) 99999 ∑ n i i=1 Py2 := Py2 = 0.553 100000 → n := (Y1 30) 99999 ∑ n i i=1 Py3 := Py3 = 0.2 100000 PY(y) := PY ← Py1 if y 10 PY ← Py2 if y 20 PY ← Py3 if y 30 The marginal PMF of runoff is plotted as follows. 1 PY(
y)0.5 0 0 20 40 y (e) → n := (Y1 10 ∧ X1 2) → m := (X1 2) 99999 ∑ Py1 := n i i=1 Py1 = 0.296 99999 ∑ m i i=1 → n := (Y1 20 ∧ X1 2) → m := (X1 2) 99999 ∑ Py2 := n i i=1 Py2 = 0.506 99999 ∑ m i i=1 → n := (Y1 30 ∧ X1 2) → m := (X1 2) 99999 ∑ Py3 := n i i=1 Py3 = 0.198 99999 ∑ m i i=1 PY(y) := PY ← Py1 if y 10 PY ← Py2 if y 20 PY ← Py3 if y
30 The conditional PMF of runoff with precipitation = 2 inches is shown as follows. 1 PY(y)0.5 0 0 20 40 y (f) Stdev (X1) = 0.678 Stdev(Y1) = 7.671 cvar(X1, Y1) = 2.701 ρ := cvar (X1, Y1) stdev (X1) · stdev (Y1) ρ = 0.519 Or directly by corr(X1, Y1) = 0.519 Therefore, the correlation coefficient between precipitation and runoff is 0.52. 6.1 ∑x ∧
765 = 85 ton = µ x n 9 (xi − 85) 2 366 ∑ 2 = = 45.75 s = n −1 8 (a) x = i = ∧ σ x = 45.75 = 6.7643 (b) one-sided hypothesis test Null hypothesis Ho: Alternative hypothesis HA: µ = 80 tons µ < 80 tons Without assuming known standard deviation, the estimated value of the test statistic to be t= x − 80 s/ n = 85 − 80 6.764 / 9 = 2.2176 with f = 9-1 = 8
d.o.f. we obtain the critical value of t from Table A.2. 2 at the 5% significance level to be tα = -1.8595. Therefore the value of the test statistic is 2.2176 > 1.8595 which is outside the region of rejection; hence the null hypothesis is accepted, and the capacity of the pile remain acceptable. (c) < µ x >0.98 = (x - k 0.01 σx , x + k 0.01 σx) n n 6.764 6.764
, 85 + 2.33) = (85 – 2.33 3 3 = (79.747, 90.253) (d) < µ x >0.98 sx s , x + t 0.01, 8 x) n n 6.764 6.764 , 85 + 2.8965) = (85 – 2.8965 3 3 = (x - t 0.01, 8 = (78.47, 91.53) 6.2 (a) Since x = 65, n = 50, σ = 6, a 2-sided 99% interval for the true mean is given by the limits 6 65 ± k0.005 50 = 0.99 = (65 – 2.58× 6 , 65 – 2.58× 50 6) 50 = (62.81, 67.19) (in
mph) 6 2 (b) Requiring 2.58 ≤ 1 ⇒ n ≥ (2.58×6) = 239.6304 n ⇒ n = 240 ⇒ (240 − 50) = 190 additional vehicles must be observed. Before we proceed to (c) and (d), let X J and X M denote John and Mary’s sample means, respectively. Both are approximately normal with mean µ (unknown true mean speed) and 6 standard deviation , hence their
difference n D = XJ – XM has an (approximate) normal distribution with mean = µ – µ = 0, and 2 2 ⎛ 6 ⎞ ⎛ 6 ⎞ 2 ⎢⎢ = 6 ⎢⎢ + ⎢⎢ , standard deviation = ⎢⎢ n ⎝ n ⎠ ⎝ n⎠ i.e. D ~ N(0, 6 2) n Hence (c) When n = 10, P(X J – X M > 2) = P(D > 2) D − µD 2−0 = P(>) σD 6/ 5 = 1 − Φ(0.745) ≅ 0.228 (d) When n = 100, P(D > 2) = 1−Φ (2−0) = 1 − Φ(2.357) ≅
0.0092 6 / 50 6.3 (a) The formula to use is 1 - α = [x − tα / 2,n −1 s n , x − tα / 2,n −1 s], where n α = 0.1, n = 10, x = 10000 cfs, s = 3000 cfs, and tα / 2,n −1 = t 0.05,9 = 1.833 Hence, the 2-sided 90% confidence interval for the mean annual maximum flow is [(10000 - 1739.044) cfs, (10000 + 1739.044) cfs] ≅ [8261 cfs, 11739 cfs] (b) Since the
confidence level (1 - α) is fixed at 90%, while s is assumed to stay at approximately s 3000 cfs, the “half-width” of the confidence interval, tα / 2, n −1 can be considered as a n function of the sample size n only. To make it equal to 1000 cfs, one must have t 0.05,n −1 ⇒ 3000 = 1000 n t 0.05, n −1 = 1/3, n which can be solved by trial and error. We
start with n = 20 (say), and increase the sample size until the half width is narrowed to the desired 1000cfs, i.e. until t 0.05, n −1 ≤ 0.33333.... n With reference to the following table, Sample size, n t0.05,n-1 20 21 22 23 24 25 26 27 1.7291 1.7247 1.7207 1.7171 1.7138 1.7108 1.7081 1.7056 t 0.05, n −1 n 0.387 0.376 0.367 0.358 0.350 0.342 0.335
0.328 We see that a sample size of 27 will do, hence an additional (27 – 10) = 17 years of observation will be required. 6.4 (a) Pile Test No. N = A/P 5 (b) N = ∑N i =1 1 2 3 4 5 1.507 0.907 1.136 1.070 1.149 5 i ∑ (N i − N)2 ≅ 1.154, = i =1 ≅ 0.219850903 ≅ 0.048 5 5 −1 (c) Substituting n = 5, N = 1.15392644, SN = 0.219850903 into the formula S
0.95 = N ∓ t0.025, n – 1 N , n where t0.025, n – 1 = t0.025,4 ≅ 2.776, we have (1.15392644 ∓ 0.27293719) = (0.881, 1.427) as a 95% confidence interval for the true mean of N. S N2 2 (d) With σ = 0.045 known, and assuming N is normal, to estimate µN to ±0.02 with 90% confidence would require k0.05 0.045 ≤ 0.02 ⇒ n ≥ (1.645 0.045 2) =
304.37 0.02 ⇒ n = 305, hence an additional (305 – 5) = 300 piles should be tested. (e) Since A = 15N, P(pile failure) = P(A < 12) = P(15N < 12) = P(N < 12/15) =Φ(12 / 15 − 1.154) 0.22 = Φ(−1.61) ≅ 0.0537 6.5 5 (a) Let x be the concrete strength. x = ∑x i =1 5 i = 3672, Sx = ∑ (x i =1 i − x) 2 5 −1 = 589778 , hence 4 589778 4 0.90 = 3672 ∓
t0.05, 5 – 1 , 5 where t0.05,4 ≅ 2.131846486, we have (3672 ∓ 366.09) = (3305.91, 4038.09) as a 90% confidence interval for the true mean of N. Note: our convention is that the subscript p in tp, dof always indicates the tail area to the right of tp, dof (b) If the “half-width” of the confidence interval, tα/2, 4 589778 4 is only 300 (psi) wide, it 5 implies
589778 4) = 1.746996, tα/2, 4 = 300 / (5 We need to determine the corresponding α. On the other hand, from t-distribution table we have (at 4 d.o.f.) t 0.1, 4 = 1.533, t 0.05, 4 = 2.132, (and tα/2, 4 increases as α/2 gets smaller in general) Hence we may use linear interpolation to get an approximate answer: over such a small “x” range (from t =
1.533 to t = 2.132), we treat “y” (i.e. α/2) as decreasing linearly, with slope m= 0.05 − 0.1 = − 0.083472454 2.132 − 1.533 Hence, as t goes from 1.533 to 1.746996, α/2 should decrease from 0.1 to α/2 = 0.1 + m(1.746996 – 1.533) = 0.1 – 0.083472454×0.213996232 = 0.082137209 ⇒ α = 2×0.082137209 = 0.164274419 Hence the confidence level is
1 − α = 1 − 0.164274419 ≅ 83.6% 6.6 (a) N = 30, x = 12.5 tons Assume σ = 3 tons Assume σ = 3 tons < µ x >0.99 = (x - k 0.005 σ n = (12.5 – 2.58 × , x + k 0.005 3 σ) n , 85 +12.5 + 2.58 × 30 3) 30 = (12.5 – 2.826, 12.5 – 2.826) = (9.674, 15.326) tons (b) Let n’ be the total number of observations required for estimating to ± 1 tone with 99%
confidence. Hence 2.58 × σ or 3 ≡1 n' n’ = (2.58x3)2 = 59.9 Therefore, (60 - 30) or 30 additional observations of truck weights would be required. 6.7 fH(h) = 2h α 2 −h2 eα 2 From Eq. 6.4, the likelihood function of n observations on a Rayleigh distributinon is n L= 2hi ∏α i =1 2 − hi 2 eα 2 Taking logarithm of both sides ln L = n ln 2 – 2n ln α + Σ ln hi
– ∂ ln L 2n 2Σhi =− + =0 α α3 ∂α 1 α2 Σ hi2 2 Æ From the given data, n = 10, Σ hi2 = (1.52 + 2.82 + …… 2.32) = 80.42 ∧ ∴ α = ± 2.836 n where ∑ =∑ i =1 ∧2 α = ∧ Σhi Σhi or α = ± n n 2 2 6.8 (a) x = 2.03 mg/l, s = 0.485 mg/l One-side hypothesis test that the minimum concentration DO is 2.0 mg/l . µ = 2.0 mg/l µ > 2.0 mg/l Null hypothesis Ho:
Alternative hypothesis HA: Without assuming known standard deviation, the estimated value of the test statistic to be x−2 t= 0.485 / 10 = 2.03 − 2 = 0.0196 0.485 / 10 with f = 10-1 = 9 d.o.f. we obtain the critical value of t from Table A.2. 2 at the 5% significance level to be tα =1.833. Therefore the value of the test statistic is 0.0196 0.95 s s , DO + t
0.025, 9) 10 10 0.485 0.485 , 2.03 + 2.2622) = (2.03 – 2.2622 10 10 = (DO - t 0.025, 9 = (1.683, 2.377) mg/l (c) < µ >0.95 = DO - t 0.05, 9 s 0.485 = 2.03 – 1.8331 =1.749mg/l 10 10 69.9 124.3 + 124.2 + 124.4 = 124.3 ft. 3 1 2 SL = [(124.3 – 124.3) 2 + (124.2 – 124.3) 2 + (124.4 – 124.3) 2] = 0.01 3 −1 s 0.01 = 0.0577 ft. Standard error of the
mean = L = n 3 40o 24.6'+...... + 40o 25.2' = 40o 25' (b) β = 5 1 2 Sβ = [(40o 24.6'−40o 25.0') 2 + ... + (40o 25.2'−40o 25.0') 2] = 0.135' 5 −1 sβ 0.135 = = 0.164' = 4.77 × 10− 5 radian Standard error of the mean = n 5 (a) L = (c) h = 3 ft ;σ h = 0.01 ft H = h + L tan β H = h + L tan β = 3 + 124.3 × tan 40o 25' = 108.85 ft. (d) σ H 2 = Var (h) +
Var (L)(tan β)2 + Var(β) x (L •sec2 β)2 = (0.01)2 + (0.0577)2(tan 40o25’)2 + (4.77 x 10-5)2 x (124.3 sec2 40o25’)2 = 0.0001+0.0024+0.0001 = 0.0026 σ H = 0.051 ft. (e) P (− k α / 2 -2.448, which is outside the region of rejection; hence the stated mileage is satisfied. (c) x 0.95 s s ⎞ ⎛ = ⎢ x − t0.025,9 i x , x + t0.025,9 i x ⎢10 10 ⎠ ⎝ 4.06 4.06 ⎞ ⎛ =
⎢ 37.4 − 2.262 × ,37.4 + 2.262 × ⎢ 10 10 ⎠ ⎝ = (34.50,40.30) 7.1 Median = 76.9 c.o.v. = ln (81.5 / 76.9) = 0.06 7.2 (a) The CDF of Gumbel distribution is: FX (x) = exp (− e −α (x − µ)) , −∞ < x < ∞ The standard variate is S = −α (X − µ) (With CDF of: FS (s) = exp − e − s) If x is observed with frequency of p , then its corresponding standard
variate is s = − ln (− ln(p)) Thus, the observed data can be arranged in the right table and plotted in the following figure. 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 x 69.3 76.7 72.3 72.9 73.5 74.8 75.8 75.9 76 76.1 76.4 77.1 77.4 78.2 78.2 78.4 79.3 80.8 81.8 85.2 P 0.048 0.095 0.143 0.190 0.238 0.286 0.333 0.381 0.429 0.476 0.524 0.571
0.619 0.667 0.714 0.762 0.810 0.857 0.905 0.952 s -1.113 -0.855 -0.666 -0.506 -0.361 -0.225 -0.094 0.036 0.166 0.298 0.436 0.581 0.735 0.903 1.089 1.302 1.554 1.870 2.302 3.020 90 Standard variate X 80 70 60 50 X = 3.3942S + 74.723 40 30 20 10 0 -2.000 -1.000 0.000 1.000 2.000 3.000 4.000 Observed maximum wind velocity V (b) The linear
regression of the data gives the trend line equation of X = 3.3942 S + 74.723 which gives X − 74.723 = 0.295(X − 74.723) 3.3942 Thus, α = 0.295 , µ = 72.723 S= (c) Perform Chi-Square test for Gumbel distribution Interval 65-70 70-75 75-80 80-85 Σ Observed frequency ni Theoretical frequency ei (ni − ei) 2 (ni − ei) 2 ei 5 11 3 20 0.356 7.602
8.240 2.860 20 0.415 6.770 7.617 0.020 1.164 0.891 0.924 0.007 2.986 The degree of freedom for the Gumbel distribution is f=4-1-2=1. For a significance level α = 2% , c.98,1 = 5.41 . Comparing with the Σ (ni − ei) 2 ei calculated, the Gumbel distribution is a valid model for the maximum wind velocity at the significance level of α = 2% . 7.3 (a) m
εµ m N +1 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16.0 16.1 16.6 16.8 17.0 17.3 17.8 17.9 18.1 18.4 18.6 18.8 19.1 19.4 20.1 0.0625 0.1250 0.1875 0.2500 0.3125 0.3750 0.4375 0.5000 0.5625 0.6250 0.6875 0.7500 0.8125 0.8750 0.9375 From Figs. 7.3a and 7.3b, it can be observed that both models (normal and lognormal) fit the data pretty well. (b) U =
17.8 sU2 = 1.312 ζ = ln(1 + σ2 1.312 2) ln(1) = 0.074 = + µ2 17.82 1 2 λ = ln µ − ζ 2 = ln(17.8) − 0.5 × 0.0742 = 2.876 Perform Chi-square test for normal distribution: Observed Theoretical Interval frequency ni frequency ei 15.5-16.5 16.5-17.5 17.5-18.5 18.5-19.5 19.5-20.5 ∑ 3.000 3.000 4.000 4.000 1.000 15 1.816 3.730 4.404 2.989 1.166 15 Perform
Chi-square test for lognormal distribution: Observed Theoretical Interval frequency ni frequency ei 15.5-16.5 16.5-17.5 17.5-18.5 18.5-19.5 19.5-20.5 ∑ 3.000 3.000 1.939 3.943 (ni − ei) 2 1.401 0.533 0.163 1.021 0.028 (ni − ei) 2 1.126 0.890 (ni − ei) 2 ei 0.772 0.143 0.037 0.342 0.024 1.317 2 (ni − ei) 2 ei 0.581 0.226 17.5-18.5 18.5-19.5 19.5-20.5 ∑ 4.000 4.000 1.000 15 4.316
2.778 1.133 15 0.100 1.493 0.018 0.023 0.537 0.016 1.382 The degree of freedom for the Normal and Lognormal distribution are both f=5-1-2=2. (ni − ei) 2 calculated, ei the 2 distributions are both valid for the rainfall intensity at the significance level of α = 2% . (n − ei) 2 As the ∑ i in Normal distribution is less than that of the Lognormal
distribution, the ei For a significance level α = 2% , c.98,2 = 7.99 . Comparing with the ∑ Normal distribution is superior to the Lognormal distribution in this problem. (c) Perform the K-S test as follows: F (Normal) 0.064 0.085 0.098 0.180 0.271 0.352 0.500 0.530 0.590 0.676 0.729 0.777 0.839 0.889 0.960 D F D (Normal) (Lognormal) (Lognormal) k
x S 1 15.8 0.067 0.003 0.059 0.008 2 16.0 0.133 0.048 0.081 0.052 3 16.1 0.200 0.102 0.095 0.105 4 16.6 0.267 0.086 0.184 0.083 5 17.0 0.333 0.062 0.282 0.052 6 17.3 0.400 0.048 0.366 0.034 7 17.8 0.467 0.033 0.517 0.051 8 17.9 0.533 0.003 0.547 0.014 9 18.1 0.600 0.010 0.606 0.006 10 18.4 0.667 0.010 0.688 0.022 11 18.6 0.733 0.004 0.738
0.005 12 18.8 0.800 0.023 0.783 0.017 13 19.1 0.867 0.028 0.840 0.026 14 19.4 0.933 0.045 0.886 0.047 15 20.1 1.000 0.040 0.954 0.046 max 0.102 0.105 The K-S test shows that the Normal distribution is more suitable in this case as it gives smaller maximum discrepancy. Fig. 7.3a Fig. 7.3b 7.4 (a) (b) The minimum time-to-failure = 0 hr. Mean time
to failure = 1 λ = 670 hrs. (c) Time-to-failure (hours) 0 – 400 400 – 800 800 – 1200 > 1200 Σ Observed Frequency ni 20 7 7 6 40 Theoritical Frequency ei 17.98 9.90 5.45 6.67 40 (ni-ei)2 (ni-ei)2 / ei 4.08 8.41 2.40 0.45 0.227 0.849 0.441 0.067 1.584 x2 distribution with f = 4 - 2 = 2 degrees of freedom and α = 5% significance level, c0.95, 2 = 5.99 (ni
− ei) 2 = 1.584 < 5.99 ∑ ei i =1 4 In this case Hence, the exponential distribution is a valid distribution model. (d) Perform K-S test as follows: k 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 max Where t x 0.13 0.78 2.34 3.55 8.82 14.29 29.75 39.1 54.85 62.09 84.09 85.28 121.58 124.09 151.44 163.95 216.4 298.58 380 393.37 S 0.025 0.050
0.075 0.100 0.125 0.150 0.175 0.200 0.225 0.250 0.275 0.300 0.325 0.350 0.375 0.400 0.425 0.450 0.475 0.500 F(Normal) D(Normal) k x S F(Normal) D(Normal) 0.000 0.025 21 412.03 0.525 0.462 0.063 0.001 0.049 22 470.97 0.550 0.507 0.043 0.004 0.071 23 633.98 0.575 0.615 0.040 0.005 0.095 24 646.01 0.600 0.621 0.021 0.013 0.112 25 656.04
0.625 0.627 0.002 0.021 0.129 26 658.38 0.650 0.628 0.022 0.044 0.131 27 672.87 0.675 0.636 0.039 0.057 0.143 28 678.13 0.700 0.639 0.061 0.079 0.146 29 735.89 0.725 0.669 0.056 0.089 0.161 30 813 0.750 0.705 0.045 0.119 0.156 31 855.95 0.775 0.724 0.051 0.120 0.180 32 861.93 0.800 0.726 0.074 0.167 0.158 33 862.93 0.825 0.727 0.098
0.170 0.180 34 895.8 0.850 0.740 0.110 0.204 0.171 35 952.65 0.875 0.761 0.114 0.218 0.182 36 1057.57 0.900 0.796 0.104 0.278 0.147 37 1296.93 0.925 0.858 0.067 0.362 0.088 38 1407.52 0.950 0.880 0.070 0.435 0.040 39 1885.22 0.975 0.941 0.034 0.446 0.054 40 2959.47 1.000 0.988 0.012 0.182 = 541.19 st2 = 359937.2 The sample size is 40.
At the 5% significance level, 0.05 D40 = 0.21 . As the maximum discrepancy is smaller than the critical value, the exponential distribution is verified. 7.5 (a) From the observation data we can get v = 1.08 , sv2 = 1.243 (b) Total no. Nos. of Observed of veh. veh/min Freq. Observed ni 0 6 0 1 8 8 6 2x3 + 3x2 ≥2 + 4x1 = 16 Σ20 Σ24 ν= Theoretical
Probability (v = 1.2) 0.301 0.361 0.338 Theoretical Frequency ei 6.02 7.22 6.76 Σ1.0 Σ20.0 24 = 1.2 20 (ni-ei)2 (ni-ei)2 / ei 0.0004 0.6084 0.5776 0.0001 0.0843 0.0854 x2 distribution with f = 3-2 = 1 degree of freedom and α = 1% significance level, c0.99,1 = 6.635 (n i − ei) 2 = 0.1698 < 6.635 ∑ ei i =1 3 In this case Hence, the Poisson distribution
is a valid distribution model. Σ0.1698 7.6 (a) f X (x) = S= x α2 e 1 x − ( )2 2 α ≥0 X α dX =α dS X = Sα , So f S (s) = f X (sα) = s ·e =0 FS (s) = 1 − e 1 − s2 2 1 − s2 dX = s ·e 2 dS ; s≥0 ; s 0.250 Σ Observed frequency ni 12 20 23 15 13 83 Theoretical frequency ei 11.09 19.06 24.56 18.25 10.04 83 (ni-ei)2 (ni-ei)2 / ei 0.8281 0.8836 2.4336 10.5625
8.7616 0.075 0.046 0.099 0.579 0.873 1.672 x2 distribution with f = 5-3 = 2 degree of freedom and α = 5% significance level, c0.95,2 = 5.991 (n i − ei) 2 In this case ∑ = 1.672 < 5.991 ei i =1 5 So the normal distribution is acceptable. 7.8 (a) X −a dX ; X = Sr + a; =r r dS dX 2 So f s (s) = f x (sr + a) = 2 (sr) · r dS r = 2s ; 0 ≤ s ≤ 1 S= =0 (b) ;
elsewhere FS(s) = S2 ; 0 ≤ s ≤ 1 = 1 ; S > 1.0 =0 ; S c.95,2=5.99 The degree of freedom for the normal distribution is f=5-1-2=2. (ni − ei) 2 calculated, The For a significance level α=5%, c.95,2=5.99. Comparing with the ∑ ei lognormal distribution is not a valid model for the ratio of settlement. Perform Anderson-Darling test for lognormal
distribution, i xi F(xi) F(xn+1-i) (2i − 1) {ln FX (xi) + ln[1 − FX (xn+1−i)]} / n 1 2 3 4 0.12 0.52 0.82 0.84 0.000 0.083 0.395 0.418 0.990 0.843 0.734 0.706 -0.783 -0.521 -0.451 -0.587 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 0.86 0.86 0.87 0.88 0.92 0.94 0.94 0.97 0.99 0.99 1.00 1.01 1.02 1.04 1.04 1.06 1.09 1.14 1.18 1.38 2.37
0.440 0.440 0.452 0.463 0.506 0.527 0.527 0.558 0.577 0.577 0.587 0.596 0.606 0.624 0.624 0.641 0.667 0.706 0.734 0.843 0.990 0.667 0.641 0.624 0.624 0.606 0.596 0.587 0.577 0.577 0.577 0.558 0.527 0.527 0.506 0.463 0.452 0.440 0.440 0.418 0.395 0.083 0.000 -0.691 -0.812 -0.922 -1.049 -1.096 -1.176 -1.281 -1.330 -1.410 -1.474 -1.487 -1.570 -1.593
-1.530 -1.587 -1.598 -1.617 -1.530 -1.461 -0.484 -0.019 Σ=-28.06 Calculate the Anderson-Darling (A-D) statistic n A2 = −∑ ⎡⎣(2i − 1) {ln FX (xi) + ln[1 − FX (xn +1−i)]} / n ⎤⎦ − n = 28.06-25 = 3.06 i =1 For lognormal distribution (lognormal distribution should be the same as normal), the adjusted A-D statistic for a sample size n=25 is, ⎛ ⎞ A* = A2
⎢1.0 + 0.75 2.25 ⎢ + 2 ⎢ =3.16 n n ⎠ The critical value cα is given by, ⎛ b b ⎞ cα = aα ⎢ 1 + 0 + 12 ⎢n n ⎠ ⎞ = 0.726 for a prescribed significance level α = 5%, aα =0.7514, b0 =-0.795, b1 =-0.89 (Table A.6). As A* is greater than cα , the lognormal distribution is not acceptable at the significance level α = 5%. 7.13 Plot data on (shifted) exponential
probability paper, 100.0 Observed Rainfall Intensity, in 90.0 y = 27.972x - 0.2884 80.0 70.0 60.0 50.0 40.0 30.0 20.0 10.0 0.0 0.00 0.50 1.00 1.50 2.00 2.50 3.00 3.50 4.00 4.50 5.00 Standard Normal Variate, S = a = -0.2884, λ = 1/27.972=0.03575 Perform Chi-square test for shifted exponential distribution, Mean depth (m) Observed frequency ni
Theoretical frequency ei 9.5 Σ Observed Frequency ni Theoritical Frequency ei 6 34 132 179 218 183 146 69 33 1000 15.00 40.91 100.33 177.35 222.07 208.57 138.96 66.75 30.06 1000 (ni-ei)2 (ni-ei)2 / ei 81.00 47.75 1002.99 2.72 16.56 653.82 49.56 5.06 8.64 5.4 1.167 9.997 0.015 0.075 3.135 0.357 0.076 0.287 20.509 x2 distribution with f = 9-3 =
6 degree of freedom and α = 1% significance level, c0.99,6 = 16.812 (n i − ei) 2 = 20.509 > 16.812 ∑ ei i =1 9 In this case So normal distribution is not a suitable one. 8.1 (a) B = stress = y / 0.1; A = strain = x / 10 ^ ^ Let, E(B A = a) = α + β a Nos. 1 2 3 4 5 6 Σ a= ^ β= bi 10 20 30 40 50 60 210 ai 9x10-4 20x10-4 28x10-4 41x10-4 52x10-4 63x10-4
213x10-4 213 × 10 −4 = 35.5 × 10 − 4 , 6 b= aibi 9x10-3 40x10-3 84x10-3 164x10-3 260x10-3 378x10-3 935x10-3 210 = 35 6 ∑ a b − nab = 935 ×10 − 6 × 35 × 35.5 ×10 ∑ a − na 9619 ×10 − 6 × 35.5 ×10 −3 i ai2 81x10-8 400x10-8 784x10-8 1681x10-8 2704x10-8 3969 x10-8 9619x10-8 −4 i −8 2 2 2 −8 i = 9.21 x 103 k/in2 ^ ^ α = b − β a = 35 −
9.21× 10 3 × 35.5 ×10 − 4 = 35 − 32.7 = 2.3 ^ So Young’s modulus = β = 9.21 x 103 k/in2 ^ (b) Assume E(B A = a) = β a 6 Δ2 = ∑ (bi − βa i) 2 i =1 Now, 6 ∂Δ = ∑ 2(bi − β a i)(−a i) = 0 ∂β i =1 or, ^ β= ∑a b ∑a i i 2 i = 935 × 10 −3 = 9.72 × 10 3 ksi −8 9619 ×10 bi2 100 400 900 1600 2500 3600 9100 8.2 (a) Plot of stopping distance vs speed of
travel Stopping Distance, in m 50 45 40 35 30 25 20 15 10 5 0 0 20 40 60 80 100 120 140 Speed, in kph (b) Vehicle No. Speed Stopping Distance (kph) (m) Xi Yi Xi2 Yi2 XiYi Yi'=a+bXi (Yi-Yi')2 1 40 15 1600 225 600 13.43 2.461 2 9 2 81 4 18 1.46 0.288 3 100 40 10000 1600 4000 36.59 11.598 4 50 15 2500 225 750 17.29 5.252 5 15 4 225 16 60 3.78
0.048 6 65 25 4225 625 1625 23.08 3.676 7 25 5 625 25 125 7.64 6.972 8 60 25 3600 625 1500 21.15 14.804 9 95 30 9025 900 2850 34.66 21.755 10 65 24 4225 576 1560 23.08 0.842 11 30 8 900 64 240 9.57 2.467 12 Total: 125 45 15625 2025 5625 679 238 52631 6910 18953 46.25 1.552 71.716 On the basis of calculations in the above table we
obtain the respective sample means of X and Y as, X = 679/12 = 56.6 kph, Y = 238/12 = 19.8 m and corresponding sample variances, 1 (52631 − 12 × 56.6 2) = 1289.84 11 1 S y2 = (6910 − 12 × 19.8 2) = 200.50 11 S x2 = , From Eq. 8.4 & 8.3, we also obtain, β= 18953 − 12 × 56.6 × 19.8 = 0.388 , 52631 − 12 × 56.6 2 α = 19.8 – 0.388×56.6 =
-2.161 From Eq. 8.6a, the conditional variance is, S Y2| X = 1 n (y i − y i') 2 = 71.716 / (12 – 2) = 7.172 ∑ n − 2 i =1 and the corresponding conditional standard deviation is S Y | x = 2.678 m From Eq. 8.9, the correlation coefficient is, n 1 ρ^ = n −1 ∑x y i =1 i i − nx y sx sy = 1 18953 − 12 × 56.6 × 19.8 = 0.98 11 1289.84 · 200.50 (c) To determine
the 90% confidence interval, let us use the following selected values of Xi = 9, 30, 60 and 125, and with t0.95,10 = 1.812 from Table A.3, we obtain, At Xi = 9; < µY X > 0.90 = 1.33 ± 1.812 × 2.679 (9 − 56.6) 2 1 + = (−1.063 → 3.723)m 12 (52631 − 12 × 56.6 2) At Xi = 30; < µY X > 0.90 = 9.48 ± 1.812 × 2.679 (30 − 56.6) 2 1 + = (7.708 →
11.252)m 12 (52631 − 12 × 56.6 2) At Xi = 60; < µY X (60 − 56.6) 2 1 > 0.90 = 21.12 ± 1.812 × 2.679 + = (19.712 → 22.528)m 12 (52631 − 12 × 56.6 2) At Xi = 125; < µY X > 0.90 = 46.34 ± 1.812 × 2.679 (125 − 56.6) 2 1 + = (43.220 → 49.460)m 12 (52631 − 12 × 56.6 2) 8.3 (a) Plot of peak hour traffic vs daily traffic 1.6 1.4 Peak hour traffic
(1000 vehicles) 1.2 1 0.8 0.6 0.4 0.2 0 0 1 2 3 4 5 6 7 Daily traffic (1000 vehicles) (b) Let X be the daily traffic volume and Y the peak hour traffic volume, both in thousand vehicles. By Eq. 8.9, the correlation coefficient between X and Y is estimated by n 1 ρ^ = n −1 ∑x y i =1 i i − nx y sx s y n With n = 5, ∑x y i =1 i i = 25.54, x = 1.02, y = 4.72, and
the sample standard deviations sx = 0.303315018 while sy = 1.251798706, ⇒ ρ^ = (1/4)(1.468 / 0.379689347) ≅ 0.967 (c) xi 5 4.5 6.5 4.6 yi 1.2 1 1.4 0.9 xiyi 6 4.5 9.1 4.14 xi2 y i' = α^ + β^xi (yi – yi’)2 25 20.25 42.25 21.16 1.085577537 0.968474793 1.436885769 0.991895341 0.0130925 0.00099384 0.00136056 0.00844475 Sum Mean 3 0.6 1.8 9
23.6 4.72 5.1 1.02 25.54 117.66 0.61716656 0.00029469 0.02418634 Since we know the total (daily) traffic count (X), and want a probability estimate about the peak hour traffic (Y), we need the regression of Y on X, i.e. the best estimates of α and β in the model E(Y | X = x) = α + βx From Eq. 8.4 & 8.3 n β= ∑x y i i =1 n ∑x i =1 2 i i − nx y − nx 2 =
(25.54 - 5×4.72×1.02) / (117.66 – 5×4.722) = 1.468 / 6.268 ≅ 0.234, and α = y − β^ x = 1.02 – 0.23420549×4.72 ≅ – 0.0854 Also, by (7.5), using the results in the table above, we estimate the variance of Y (assumed constant) S Y2| x = 1 n (y i − y i') 2 ∑ n − 2 i =1 = 0.02418634 / (5 – 2) ≅ 0.0081 Hence when the daily traffic is 6000 vehicles, i.e. x =
6, Y has the estimated parameters µY = –0.0854 + 0.234×6 = 1.319 σY = 0.0080621140.5 = 0.089789278 Hence the estimated probability of peak hour traffic exceeding 1500 vehicles is P(Y > 1.5) = 1 – P(Y ≤ 1.5) = 1 – Φ (1.5 − 1.319) = 1 – Φ(2.01) 0.09 ≅ 0.0222 8.4 (a) Plot of per capita energy consumption vs per capita GNP 5000 Per Capita
Energy Consumption 4500 4000 3500 Y 3000 Y' 2500 lower 2000 upper 1500 1000 500 0 0 2000 4000 6000 8000 10000 12000 Per Captia GNP (b) Country # Per Capita GNP Per Capita Energy Consumption Xi Yi Xi2 Yi2 XiYi Yi'=a+bXi (Yi-Yi')2 1 600 1000 360000 1000000 600000 950.37 2463.269 2 2700 700 7290000 490000 1890000 1535.64
698286.734 3 2900 1400 8410000 1960000 4060000 1591.38 36624.478 4 4200 2000 17640000 4000000 8400000 1953.68 2145.238 5 3100 2500 9610000 6250000 7750000 1647.11 727412.944 6 5400 2700 29160000 7290000 14580000 2288.12 169643.906 7 8600 2500 73960000 6250000 21500000 3179.96 462341.123 8 10300 4000
106090000 16000000 41200000 Total: 37800 16800 252520000 43240000 99980000 3653.74 On the basis of calculations in the above table we obtain the respective sample means of X and Y as, X = 37800/8 = 4725, Y = 16800/8 = 2100 and corresponding sample variances, 1 (252520000 − 8 × 4725 2) = 10559285.71 , 7 1 S y2 = (43240000 − 8 ×
2100 2) = 1137142.86 7 S x2 = From Eq. 8.4 & 8.3, we also obtain, β= (c) 99980000 − 8 × 4725 × 2100 = 0.279 , α = 2100 – 0.279×4725 = 781.725 2 252520000 − 8 × 4725 From Eq. 8.9, the correlation coefficient is, n 1 ρ^ = n −1 (d) ∑x y i =1 i i − nx y sx sy = 1 99980000 − 8 × 4725 × 2100 = 0.85 7 10559285.71 · 1137142.86 From Eq. 8.6a,
the conditional variance is, S Y2| X = 1 n (y i − y i') 2 = 2218817.515 / 6 = 369801.799 ∑ n − 2 i =1 and the corresponding conditional standard deviation is SY | X = 608.113 (e) To determine the 95% confidence interval, let us use the following selected values of Xi = 600, 5400 and 10300, and with t0.975,6 = 2.447 from Table A.3, we obtain, At Xi =
600; 119893.103 2218810.796 < µY X > 0.95 = 950.37 ± 2.447 × 608.113 (600 − 4725) 2 1 + = (62.263 → 1835.997) 8 (252520000 − 8 × 4725 2) At Xi = 5400; < µY X (5400 − 4725) 2 1 > 0.95 = 2288.12 ± 2.447 × 608.113 + = (1749.407 → 2827.253) 8 (252520000 − 8 × 4725 2) At Xi = 10300; < µY (f) X > 0.95 = 3653.74 ± 2.447 × 608.113
(10300 − 4725) 2 1 + = (2556.391 → 4754.469) 8 (252520000 − 8 × 4725 2) Similarly, β = 2.588 , α = -709.673 and S X |Y = 1853.080 for predicting the per capita GNP on the basis of the per capita energy consumption. 8.5 (a) Let X be the car weight in kips; X ~ N(3.33, 1.04). Hence X − µ 4.5 − 3.33) P(X > 4.5) = P(> σ 1.04 = P(Z > 1.125) = 1 –
Φ(1.125) = 1 - 0.8697 ≅ 0.130 (b) Let Y be the gasoline mileage. For linear regression, we assume E(Y | X = x) = α + βx and seek the best (in a least squares sense) estimates of α and β in the model. Based on the following data, sum average xi (kips) 2.5 4.2 3.6 3.0 yi (mpg) 25 17 20 21 13.3 3.325 83.0 20.75 n β= ∑x y i i =1 n ∑x i =1 2 i i xiyi xi2 y i' =
α^ + β^xi (yi – yi’)2 6.25 17.64 12.96 9.00 62.5 71.4 72 63 24.33641 16.94624 19.55453 22.16283 0.440358 0.002891 0.198442 1.352165 45.9 268.9 1.993856 − nx y − nx 2 = (45.9 - 4×3.325×20.75) / (268.9 – 4×3.3252) = 1.468 / 6.268 ≅ - 4.35, and α = y − β^ x = 20.75 – 4.347158218×3.325 ≅ 35.20 Also, by Eq. 8.6a, using the results in the
preceding table, we estimate the variance of Y (which is assumed constant) S Y2| x = 1 n ∑ (yi − yi') 2 n − 2 i =1 = 1.993856 / (4 − 2) ≅ 0.997 Hence when the car weighs 2.3 kips, i.e. x = 2.3, Y has the estimated parameters µY = 35.20430108 + (-4.347158218)×2.3 = 25.206 σY = 0.9969278030.5 = 0.998 Hence the estimated probability of gas
mileage being more than 28 mpg is P(Y > 28) = 1 – P(Y ≤ 28) = 1 – Φ (28 − 25.206) 0.998 = 1 – Φ(2.8) = 1 – 0.9974 ≅ 0.0026 To determine the 95% confidence interval, let us use the following selected values of Xi = 2.5 and 4.2, and with t0.975,2 = 4.303 from Table A.3, we obtain, At Xi = 2.5; (c) < µY X > 0.95 = (2.5 − 3.3) 2 1 + = (21.215 → 27.465)mpg 4
(45.85 − 4 × 3.3 2) > 0.95 = 24.34 ± 4.303 × 0.998 At Xi = 30; < µY X > 0.95 = 16.95 ± 4.303 × 0.998 (4.2 − 3.3) 2 1 + = (13.613 → 20.287)mpg 4 (45.85 − 4 × 3.3 2) Plot of gasoline mileage vs weight 30 Gasoline Mileage (mpg) 25 Y 20 Y' lower 15 upper 10 5 0 0 1 2 3 Weight (kip) 4 5 8.6 (a) Let Y be the number of years of experience, and M be
the measurement error in inches. We have the following data: i 1 2 3 4 5 Σ mi 1.5 0.8 1 0.8 0.5 4.6 yi 3 5 10 20 25 63 yimi 4.5 4 10 16 12.5 47 yi2 9 25 100 400 625 1159 From Eq. 8.4 & 8.3, we have β = ∑ y m − nym = -0.030010953, and ∑ y − ny i i 2 i 2 α = m − βy = 1.298138007 so our regression model is Mmean = 1.298138007 - 0.030010953Y,
Hence the mean measurement error when a surveyor has 15 years of experience is estimated to be 1.298138007 + (-0.030010953)(15) = 0.847973713, while the estimated variance of M is 0.073026652 (by equation 7.5), hence P(M < 1 | Y = 15) = Φ(1 − 0.847973713) 0.073026652 = Φ(0.562571845) ≅ 0.713 (b) No, our data for Y ranges from 3 to
25, so our regression model is only valid for predictions using Y values within this range. 60 is outside the range, where our model may not be correct. In fact, other factors could come into play, such as effects of aging—a 100-year-old surveyor might not be able to see the crosshair anymore! 8.7 (a) Let E(DO T=t) = α + βt T DO (days) (ppm) Xi Yi Xi2
Yi2 XiYi Yi'=a+bXi (Yi-Yi')2 1 0.5 0.28 0.25 0.0784 0.14 0.29 0.00012 2 1 0.29 1 0.0841 0.29 0.27 0.00052 3 1.6 0.29 2.56 0.0841 0.464 0.24 0.00260 4 1.8 0.18 3.24 0.0324 0.324 0.23 0.00246 5 2.6 0.17 6.76 0.0289 0.442 0.19 0.00048 6 3.2 0.18 10.24 0.0324 0.576 0.16 0.00027 7 3.8 0.1 14.44 0.01 0.38 0.14 0.00124 8 4.7 0.12 22.09 0.0144 0.564
0.09 0.00074 Total: 19.2 1.61 60.58 0.3647 3.18 0.00842 Σ ti = 19.2, Σ DOi = 1.61, Σ ti DOi = 3.180, Σ ti2 = 60.58, Σ DOi2 = 0.3647, Δ2 = Σ(DOi – DOi’)2 = 84.825 x 10-4 t= ^ 19.2 = 2.4, 8 β= DO = Σt i · DOi − nt DO ^ Σt i − nt 2 2 = 1.61 = 0.20 8 3.18 − 8 × 2.4 × 0.20 = −0.0455 60.58 − 8 × 2.4 2 ^ α = DO − β t = 0.20 + 0.0455 × 2.4 = 0.3092
So the least-squares regression equation is, ^ E (DO T=t) = 0.3092 − 0.0455 t (b) S x2 = 1 (60.58 − 8 × 2.4 2) = 2.0714 , 7 S y2 = 1 (0.3647 − 8 × 0.2 2) = 0.0058 7 n 1 ρ^ = n −1 S Y2| X = ∑x y i =1 i i − nx y sx sy = 1 3.18 − 8 × 2.4 × 0.2 = −0.89 7 2.0714 · 0.0058 1 n (y i − y i') 2 = 0.00842 / 6 = 0.0014 ∑ n − 2 i =1 and the corresponding
conditional standard deviation is SY | X = 0.037 ppm. 8.8 Cost vs floor area 250 thousand $ 200 150 100 50 0 0 (a) 1 2 3 1000 sq ft The standard deviation of Y is 4 0.0025 = 0.05. When X = 0.35, the mean of Y is E(Y | X = 0.35) = 1.12×0.35 + 0.05 = 0.442, hence P(Y > 0.3 | X = 0.35) = 1 – P(Y ≤ 0.3 | X = 0.35) 0.3 − 0.442) = 0.9977 = 1 – Φ(0.05
Also, as preparation for part (b), if X = 0.4 ⇒ E(Y | X = 0.4) = 1.12×0.4 + 0.05 = 0.498 ⇒ P(Y > 0.3 | X = 0.4) = 1 – P(Y ≤ 0.3 | X = 0.4) 0.3 − 0.498) = 1 – Φ(0.05 = 0.999963 (b) Theorem of total probability gives P(Y > 0.3) = P(Y > 0.3 | X = 0.35)P(X = 0.35) + P(Y > 0.3 | X = 0.4)P(X = 0.4) = 0.9977×(1/5) + 0.999963×(4/5) ≅ 0.9995 (c) Let YA and
YB be the respective actual strengths at A and B. Since these are both normal, YA ~ N(0.442, 0.05); YB ~ N(0.498, 0.05), Hence the difference D = YA – YB ~ N(0.442 – 0.498, D ~ N(−0.056, 0.005) Hence P(YA > YB) = P(YA – YB > 0) = P(D > 0) 0.05 2 + 0.05 2), i.e. = 1 − Φ[0 − (−0.056)] 0.005 = 1 − Φ(0.791959595) = 1 − 0.785807948 ≅ 0.214 8.9
(a) The formulas to use are n ∑x y β= i i =1 n ∑x i =1 2 i − nx y i − nx 2 and α = y − β^ x The various quantities involved are calculated in the following table: n= 3 xi yi 1.05 63 1.83 93 3.14 204 Sum = 6.02 359 Mean = 2.00666667 119.666667 Beta = Alpha = 154.676667 119.666667 xiyi xi2 66.15 168.36 640.56 875.07 1.1025 3.3489 9.8596 14.311
/ - 2.23086667 139.131807 yi' (yi-yi')2 53.3363865 93.3854267 107.417521 237.699949 198.246093 33.1074492 364.192825 = 69.335 -19.465 The regression line y = –19.46514060 + 69.33478768 x can be plotted on the scattergram, and has the best fit to the data (in a least square sense). (b) Note the words “for given floor area”. In this case, we
should not simply calculate the sample standard deviation of the y data, since although the individual yi’s may deviate a lot from their mean, this may be due to changes in x and hence not really a randomness of y. Hence we need to calculate the conditional variance E(Y | x), assuming it is a constant. An unbiased estimate of this is given by Eq. 8.6a as
S Y2| x = 1 n ∑ (yi − yi') 2 n − 2 i =1 = 364.192825 / (3 − 2) ≅ 364.193 Thus the standard deviation of construction cost for given floor area is estimated as SY|x = 364.1928250.5 ≅ 19.08 (c) The individual sample standard deviations are sx = 1.056140773, sy = 74.46028024. Plugging these (and beta) into 8.10a, ρ^ = β^ Sx Sy = 69.3347877
×1.056140773÷74.46028024 ≅ 0.983 Since it is close to 1, there is a strong linear relationship between X and Y (d) When X = 2.5, µY = α + β(25) = −19.46514060 + 69.33478768(25) ≅ 153.8718286, thus Y ~ N(153.8718286, 19.08383675) ⇒ P(Y < 180 | X = 2.5) = P(Z < 180 − 153.871828 6) = Φ(1.369125704) ≅ 0.915 19.0838367 5 8.10 (a) Car #
Rated Actual Mileage Mileage (mpg) (mpg) Xi Yi Xi2 Yi2 XiYi Yi'=a+bXi (Yi-Yi')2 1 20 16 400 256 320 15.54 0.215 2 25 19 625 361 475 19.29 0.086 3 30 25 900 625 750 23.05 3.807 4 30 22 900 484 660 23.05 1.100 5 25 18 625 324 450 19.29 1.671 6 15 12 225 144 180 11.78 0.048 Total: 145 112 3675 2194 2835 6.927 On the basis of calculations in
the above table we obtain the respective sample means of X and Y as, X =145/6 = 24.2, Y = 112/6 = 18.7 and corresponding sample variances, S x2 = 1 (3675 − 6 × 24.2 2) = 34.17 , 5 1 S y2 = (2194 − 6 × 18.7 2) = 20.67 5 From Eq. 8.4 & 8.3, we also obtain, β= (b) 2835 − 6 × 24.2 × 18.7 = 0.751 , α = 18.7 – 0.751×24.2 = 0.512 2 3675 − 6 ×
24.2 From Eq. 8.9, the correlation coefficient is, n 1 ρ^ = n −1 ∑x y i =1 i i − nx y sx sy = 1 2835 − 6 × 24.2 ×18.7 = 0.97 5 34.17 · 20.67 From Eq. 8.6a, the conditional variance is, S Y2| X = 1 n ∑ (y i − y i') 2 = 6.927 / 4 = 1.732 n − 2 i =1 and the corresponding conditional standard deviation is SY | X = 1.316 mpg. (c) YQ = actual milage of model
Q car YR = actual milage of model R car ∵ XQ = rated milage of model Q car = 22 mpg E(YQ) = 0.512 + 0.751x22 = 17.03 Similarly E(YR) = 0.512 + 0.751x24 = 18.54 Hence YQ = N(17.03, 1.32); YR = (18.54, 1.32) Assume YQ, YR to be statistically independent, P(YQ > YR) = P(YR - YQ < 0) = Φ − (18.54 − 17.03) 1.32 2 = Φ (−0.809) = 0.21 (d) To
determine the 90% confidence interval, let us use the following selected values of Xi = 15, 20 and 30, and with t0.95,4 = 2.132 from Table A.3, we obtain, At Xi = 15; < µY X > 0.90 = 11.78 ± 2.132 ×1.316 1 (15 − 24.2) 2 + = (9.446 → 14.114) mpg 6 (3675 − 6 × 24.2 2) At Xi =20; < µY X (20 − 24.2) 2 1 > 0.90 = 15.54 ± 2.132 ×1.316 + = (14.066 →
17.014) mpg 6 (3675 − 6 × 24.2 2) At Xi = 30; < µY X > 0.90 = 23.05 ± 2.132 ×1.316 (30 − 24.2) 2 1 + = (21.331 → 24.769) mpg 6 (3675 − 6 × 24.2 2) 8.11 (a) Case No. Obs. S'mt. Cal. S'mt. Yi Xi Yi2 Xi2 XiYi Xi'=a+bYi (Xi-Xi')2 1 0.7 4.8 0.49 23.04 3.36 -0.93 32.864 2 64 25.1 4096 630.01 1606.4 55.01 894.691 3 5 3.8 25 14.44 19 2.87 0.869 4 25
26.4 625 696.96 660 20.54 34.299 5 29.5 27.8 870.25 772.84 820.1 24.52 10.755 6 3.8 0.5 14.44 0.25 1.9 1.81 1.708 7 5.9 6.5 34.81 42.25 38.35 3.66 8.048 8 38.1 2.6 1451.61 6.76 99.06 32.12 871.498 9 185 174 34225 30276 32190 161.95 145.195 10 28.1 26.7 789.61 712.89 750.27 23.28 11.674 11 0.6 0.6 0.36 0.36 0.36 -1.02 2.628 12 29.2 31.5
852.64 992.25 919.8 24.26 52.484 13 32 31 1024 961 992 26.73 18.233 14 5.4 3.7 29.16 13.69 19.98 3.22 0.229 15 3.6 4 12.96 16 14.4 1.63 5.615 16 35.9 25.9 1288.81 670.81 929.81 30.18 18.291 17 11.6 11.3 134.56 127.69 131.08 8.70 6.757 18 12.7 14.2 161.29 201.64 180.34 9.67 20.495 19 46 42.1 2116 1772.41 1936.6 39.10 8.981 20 3.8 1.58
14.44 2.4964 6.004 1.81 0.052 21 4.9 5 24.01 25 24.5 2.78 4.932 22 11.7 11.8 136.89 139.24 138.06 8.79 9.066 23 1.83 3.39 3.3489 11.4921 6.2037 0.07 11.049 24 9.43 3.24 88.9249 10.4976 30.5532 6.78 12.552 25 6.6 4.3 43.56 18.49 28.38 4.28 0.000 Total: 600.4 491.8 48063.16 38138.51 41546.51 2182.967 On the basis of calculations in the
above table we obtain the respective sample means of X and Y as, Y = 600.4/25 = 24, X = 491.8/25 = 19.7 and corresponding sample variances, 1 (48063.16 − 25 × 24 2) = 1401.91 24 1 S X2 = (38138.51 − 25 ×19.7 2) = 1185.98 24 S Y2 = , From Eq. 8.4 & 8.3, we also obtain, β= 41546.51 − 25 × 24 × 19.7 = 0.884 , 48063.16 − 25 × 24 2 α =
19.7 – 0.884×24 = -1.551 Hence, E(X | y) = α + βy = -1.551 + 0.884y From Eq. 8.6a, the conditional variance is, S X2 |Y = 1 n (x i − x i') 2 = 2182.967 / (25 − 2) = 94.912 ∑ n − 2 i =1 and the corresponding conditional standard deviation is S X |Y = 9.742 (b) From Eq. 8.9, the correlation coefficient is, n 1 ρ^ = n −1 (c) ∑x y i =1 i i − nx y sx sy = 1
41546.51 − 25 ×19.7 × 24 = 0.96 24 1185.98 · 1401.91 To determine the 95% confidence interval, let us use the following selected values of Yi = 64, 25, 5.9, 185, 0.6, 3.6, 11.6 and 46, and with t0.975,23 = 2.069 from Table A.3, we obtain, At Yi = 64; < µ X Y > 0.95 = 55.01 ± 2.069 × 9.742 (64 − 24) 2 1 + = (49.048 → 60.975) 25 (48063.16 − 25 ×
24 2) At Yi = 25; < µX Y (25 − 24) 2 1 > 0.95 = 20.54 ± 2.069 × 9.742 + = (16.511 → 24.576) 25 (48063.16 − 25 × 24 2) At Yi = 5.9; < µ X Y > 0.95 = 3.66 ± 2.069 × 9.742 (5.9 − 24) 2 1 + = (−0.833 → 8.159) 25 (48063.16 − 25 × 24 2) At Yi = 185; < µ X Y > 0.95 = 161.95 ± 2.069 × 9.742 (185 − 24) 2 1 + = (143.806 → 180.094) 25 (48063.16 −
25 × 24 2) At Yi = 0.6; < µX Y (0.6 − 24) 2 1 > 0.95 = −1.02 ± 2.069 × 9.742 + = (−5.804 → 3.761) 25 (48063.16 − 25 × 24 2) At Yi = 3.6; < µ X Y > 0.95 = 1.63 ± 2.069 × 9.742 (3.6 − 24) 2 1 + = (−2.983 → 6.244) 25 (48063.16 − 25 × 24 2) At Yi = 11.6; < µX Y (11.6 − 24) 2 1 > 0.95 = 8.70 ± 2.069 × 9.742 + = (4.445 → 12.957) 25 (48063.16 −
25 × 24 2) At Yi = 46; < µ X Y > 0.95 = 39.10 ± 2.069 × 9.742 (46 − 24) 2 1 + = (34.403 → 43.803) 25 (48063.16 − 25 × 24 2) Plot of calculated settlement vs observed settlement 200 Cal. S'mt. 150 Y Y' lower upper 100 50 0 0 50 100 -50 Obs. S'mt. 150 200 8.12 (a) Plot of % loss in ridership vs % fare increase 16 % Loss in Ridership 14 12 Y 10 Y' 8
lower 6 upper 4 2 0 0 10 20 30 40 % Fare Increase (b) Fare Loss in Increase Ridership % % Xi Yi Xi2 Yi2 XiYi Yi'=a+bXi (Yi-Yi')2 1 5 1.5 25 2.25 7.5 2.05 0.300 2 35 12 1225 144 420 11.55 0.202 3 20 7.5 400 56.25 150 6.80 0.491 4 15 6.3 225 39.69 94.5 5.22 1.176 5 4 1.2 16 1.44 4.8 1.73 0.282 6 6 1.7 36 2.89 10.2 2.36 0.442 7 18 7.2 324 51.84 129.6
6.17 1.070 8 23 8 529 64 184 7.75 0.063 9 38 11.1 1444 123.21 421.8 12.50 1.962 10 8 3.6 64 12.96 28.8 3.00 0.362 11 12 3.7 144 13.69 44.4 4.27 0.320 12 17 6.6 289 43.56 112.2 5.85 0.564 13 17 4.4 289 19.36 74.8 5.85 2.100 14 13 4.5 169 20.25 58.5 4.58 0.007 15 7 2.8 49 7.84 19.6 2.68 0.014 16 23 8 529 64 184 7.75 0.063 Total: 261.0 90.1
5757.00 667.23 1944.70 9.417 On the basis of calculations in the above table we obtain the respective sample means of X and Y as, X =261/16 = 16.3, Y = 90.1/16 = 5.6 and corresponding sample variances, S x2 = 1 1 (5757 − 16 × 16.3 2) = 99.96 , S y2 = (667.23 − 16 × 5.6 2) = 10.66 15 15 From Eq. 8.4 & 8.3, we also obtain, β= (c) 1944.7 − 16
× 16.3 × 5.6 = 0.317 , α = 5.6 – 0.317×16.3 = 0.464 2 5757 − 16 ×16.3 From Eq. 8.9, the correlation coefficient is, n 1 ρ^ = n −1 ∑x y i =1 i i − nx y sx sy = 1 1944.7 − 16 ×16.3 × 5.6 = 0.97 15 99.96 · 10.66 From Eq. 8.6a, the conditional variance is, S Y2| X = 1 n (y i − y i') 2 = 9.417 / 14 = 0.673 ∑ n − 2 i =1 and the corresponding conditional
standard deviation is SY | X = 0.820 (d) To determine the 90% confidence interval, let us use the following selected values of Xi = 4, 23, 38, 12 and 7, and with t0.95,14 = 1.761 from Table A.3, we obtain, At Xi = 4; < µY X > 0.90 = 1.73 ± 1.761× 0.82 (4 − 16.3) 2 1 + = (1.147 → 2.315) 16 (5757 − 16 ×16.3 2) At Xi =23; < µY X (23 − 16.3) 2 1 > 0.90
= 7.75 ± 1.761× 0.82 + = (7.311 → 8.188) 16 (5757 − 16 ×16.3 2) At Xi = 38; < µY X > 0.90 = 12.5 ± 1.761× 0.82 (38 − 16.3) 2 1 + = (11.615 → 13.387) 16 (5757 − 16 ×16.3 2) > 0.90 = 4.27 ± 1.761× 0.82 (12 − 16.3) 2 1 + = (3.870 → 4.661) 16 (5757 − 16 ×16.3 2) > 0.90 = 2.68 ± 1.761× 0.82 (7 − 16.3) 2 1 + = (2.181 → 3.183) 16 (5757 − 16
×16.3 2) At Xi = 12; < µY X At Xi = 7; < µY X 8.13 (a) Given I = 6, E(D) = 10.5 + 15x6 = 100.5 Hence P(D>150) = 1 − Φ (b) (150 − 100.5) = 1 − Φ (1.65) = 1 − 0.95 = 0.05 30 Given I = 7, E(D) = 10.5 + 15x7 = 115.5 Given I = 8, E(D) = 10.5 + 15x8 = 130.5 Hence E(D) = E(D6)x0.6 + P(D7)x0.3 + E(D7)x0.1 = 100.5x0.6 + 115.5x0.3 +130.5x0.1 =
108 million $ 8.14 (a) Load Deflection tons cm Xi Yi Xi2 Yi2 XiYi Yi'=a+bXi (Yi-Yi')2 1 8.4 4.8 70.56 23.04 40.32 4.44 0.126 2 6.7 2.9 44.89 8.41 19.43 3.43 0.276 3 4.0 2.0 16 4 8 1.81 0.037 4 10.2 5.5 104.04 30.25 56.1 5.52 0.001 Total: 29.3 15.2 235.49 65.70 123.85 0.440 On the basis of calculations in the above table we obtain the respective sample
means of X and Y as, X =29.3/4 = 7.3, Y = 15.2/4 = 3.8 and corresponding sample variances, 1 S x2 = (235.49 − 4 × 7.3 2) = 6.96 , 3 1 S y2 = (65.70 − 4 × 3.8 2) = 2.65 3 From Eq. 8.4 & 8.3, we also obtain, β= 123.85 − 4 × 7.3 × 3.8 = 0.599 , 235.49 − 4 × 7.3 2 α = 3.8 – 0.599×7.3 = -0.591 From Eq. 8.6a, the conditional variance is, S Y2| X = 1
n ∑ (y i − y i') 2 = 0.44 / 2 = 0.220 n − 2 i =1 and the corresponding conditional standard deviation is SY | X = 0.469 cm. (b) To determine the 90% confidence interval, let us use the following selected values of Xi = 4 and 10.2, and with t0.95,2 = 2.92 from Table A.3, we obtain, At Xi = 4; < µY X > 0.90 = 1.81 ± 2.92 × 0.469 (4 − 7.3) 2 1 + = (0.597
→ 3.017) cm 4 (235.49 − 4 × 7.3 2) At Xi =10.2; < µY (c) X (10.2 − 7.3) 2 1 > 0.90 = 5.52 ± 2.92 × 0.469 + = (4.422 → 6.625) cm 4 (235.49 − 4 × 7.3 2) Under a load of 8 tons, E(Y) = -0.591 + 0.599x8 = 4.20 cm Let y75 be the 75-percentile reflection P(Y < y75) = 0.75 = Φ Hence y 75 − 4.20 0.469 y 75 − 4.20 = Φ −1 (0.75) = 0.675 0.469 and y 75
= 4.52cm 8.15 (a) Deformation Brinell Hardness mm kg/mm2 Xi Yi Xi2 Yi2 XiYi Yi'=a+bXi (Yi-Yi')2 1 6 68 36 4624 408 68.65 0.419 2 11 65 121 4225 715 61.88 9.720 3 13 59 169 3481 767 59.18 0.031 4 22 44 484 1936 968 47.00 9.000 5 28 37 784 1369 1036 38.88 3.543 6 35 32 1225 1024 1120 29.41 6.699 Total: 115 305 2819 16659 5014 29.412
On the basis of calculations in the above table we obtain the respective sample means of X and Y as, X =115/6 = 19.2, Y = 305/6 = 50.8 and corresponding sample variances, 1 S x2 = (2819 − 6 × 19.2 2) = 122.97 , 5 S y2 = 1 (16659 − 6 × 50.8 2) = 230.97 5 From Eq. 8.9, the correlation coefficient is, n 1 ρ^ = n −1 (b) ∑x y i =1 i i − nx y sx sy From
Eq. 8.4 & 8.3, we also obtain, = 1 5014 − 6 ×19.2 × 50.8 = −0.99 5 122.97 · 230.97 β= 5014 − 6 ×19.2 × 50.8 = −1.353 , α = 50.8 + 1.353×19.2 = 76.765 2 2819 − 6 ×19.2 From Eq. 8.6a, the conditional variance is, S Y2| X = 1 n (y i − y i') 2 = 29.412 / 4 = 7.353 ∑ n − 2 i =1 and the corresponding conditional standard deviation is SY | X = 2.712
kg/mm2. (c) E(YX=20) = 76.765 – 1.353x20 = 49.7 Hence, P(40 0.9) = 0.7 and P(p > 0.9) = 0.3 (b) f’’(p) = k × L(p) × f’(p) 3 3 L(p) = ( ) p3 × (1-p)0 = p3 0 ≤ p ≤ 0.9 0.9 ≤ p ≤ 1.0 1 ∫3 8 6 4 To find k, 0.9 10 f"(p) So, f’’(p) = k × p3 × 1/3; = k × p3 × 7; =0, elsewhere 12 k × p dp + 3 0 1.0 2 0.9 0 3 ∫ k × 7 · p dp = 1.0 0 or, p 4 0.9 p 4 1.0 → k = 1.523
k[ ] + 7k [ ] = 1.0 3 4 0 4 0.9 0.5 1 p 1 So, f’’(p) = 0.508 p3 ; = 10.663 p3 ; =0, elsewhere (c) E ''(p) = 0 ≤ p ≤ 0.9 0.9 ≤ p ≤ 1.0 0.9 1.0 0 0.9 4 4 ∫ 0.508 p dp + ∫ 10.663 p dp h i = 0.508[= 0.933 p 5 0.9 p 5 1.0] + 10.663[ ] 5 0 5 0.9 9.9 P(ν = 20) = 2/3, P(ν =15) = 1/3 (a) Let X = Number of occurrences of fire in the next year. P(X=20) = P(X=20ν
=15) P(ν =15) + P(X=20ν =20) P(ν =20) = e −15 (15) 20 1 e −20 (20) 20 2 × + × 20! 3 20! 3 = 0.0139 + 0.0592 = 0.0731 (b) P’’(ν =15) = P(X = 20ν = 15) · P(ν = 15) P(X = 20) = 0.0139 = 0.19 0.0731 Similarly, P’’(ν =20) = 0.0592 = 0.81 0.0731 (c) P(X=20) = P(X=20ν =15) P’’(ν =15) + P(X=20ν =20) P’’(ν =20) e −15 (15) 20 (20) 20 = ×
0.19 + × 0.81 20! 20! = 0.0079 + 0.0720 = 0.0799 9.10 λ is the parameter to be updated with the prior distribution of: P '(λ = 1 5) = 1/ 3 P '(λ = 1 10) = 2 / 3 (a) Let ε =accidents were reported on days 2 and 5. The probability to observe ε is P(ε) = P(ε | λ = 1/ 5) P '(λ = 1/ 5) + P(ε | λ = 1/10) P '(λ = 1/10) 1 2 1 3 1 1 2 1 3 2 = × exp(−) × × exp(−)
× + × exp(−) × × exp(−) × 5 5 5 5 3 10 10 10 10 3 −3 = 8.949 × 10 Combining the observation with the prior distribution of λ with Bayesian’s theorem, the posterior distribution of λ can be calculated as follows: 1 2 1 3 1 × exp(−) × × exp(−) × P(ε | λ = 1/ 5) P '(λ = 1/ 5) 5 5 5 5 3 = 0.548 P "(λ = 1/ 5) = −3 P(ε) 8.949 × 10 P "(λ = 1/10) = 1
− 0.548 = 0.452 (b) Let ε =no accidents will occur for at least 10 days after second accident. P(ε) = P(T > 10) = 1 − P(T ≤ 10) = 1 − P(T ≤ 10 | λ = 1/ 5) P "(λ = 1/ 5) − P(T ≤ 10 | λ = 1/10) P "(λ = 1/10) = 1 − (1 − e − 10 5) × 0.548 − (1 − e − 10 10) × 0.452 = 0.240 (c) The updated distribution of λ as prior distribution in this problem: P '(λ = 1/ 5)
= 0.548 P '(λ = 1/10) = 0.452 Let ε = after the second accident in 10th day no accident was observed until the 15th day. The probability to observe Z is: P(ε) = P(T > 5) = [1 − P(T ≤ 5 | λ = 1/ 5)]P '(λ = 1/ 5) + [P(T ≤ 5 | λ = 1/10)]P '(λ = 1/10) − 5 = [1 − (1 − e 5)] × 0.548 + [1 − (1 − e 5 10)] × 0.452 = 0.202 + 0.274 = 0.476 Combining the
observation with the prior distribution of λ with Bayesian’s theorem, the posterior distribution of λ can be calculated as follows: P "(λ = 1 5) = [1 − P(T ≤ 5 | λ = 1/ 5)]P '(λ = 1/ 5) 0.202 = 0.424 P(Z) 0.476 P "(λ = 1 10) = 1 − 0.424 = 0.575 (d) 1 1 1 1 P(n = nc) = P (n = nc | λ =) P(λ =) + P(n = nc | λ =) P(λ =) 5 5 10 10 nc nc 2 1 = × e −2 ×
0.548 + × e −1 × 0.425 nc ! nc ! = 0.074 × 2 nc 1nc + 0.156 × nc ! nc ! 0.2nc 0.449 × 1 nc ! P "(λ =) = 5 P(n = nc) 0.1nc 1 nc ! P "(λ =) = 10 P(n = nc) 0.385 × 1 5 Set P "(λ =) = P "(λ = 1) 10 We get nc = 1.076 . Since nc is an integer, nc = 2 . (e) Non-informative prior for λ is used here. Likelihood function is: L(λ) = λ e −2 λ λ e −3λ = λ 2 e
−5λ Posterior distribution: f "(λ) ∝ L(λ) = λ 2 e −5λ And as ∫ +∞ 0 λ 2e −5λ dλ = 1 +∞ Γ(3) 2 = (5λ) 2 e − (5λ)d(5λ) = ∫ 25 0 25 25 Thus the posterior distribution is: f "(λ) = 12.5λ 2 e −5λ 9.11 M is the parameter to be updated. It’s convenient to prescribe a conjugate prior to the Poisson process. From the information given in the problem, the mean
and variance of the gamma distribution of M is: E '(µ) = k' k '/ v '2 = 10 δ '(µ) = 0.4 v' k '/ v ' , Thus, k ' = 6.25 , v ' = 0.625 (a) It follows that the posterior distribution of M is also gamma distributed. From the relationship given in Table 9.1 between the prior and posterior statistics, and the sample data, the posterior distribution parameter of M
can be estimated as: k " = k '+ 1 = 7.25 v " = v '+ 0.6 = 1.225 k " 7.25 E "(M) = = = 5.918 v " 1.225 δ '(M) = k "/ v "2 = 0.371 k "/ v " (b) P(X = 0) = ∫ +∞ =∫ +∞ =∫ +∞ 0 0 0 P(X = 0 | m) f '(m)dm (0.4m)0 −0.4 m v "(v " m) k " −1 − mv " ×e × × e dm 0! Γ(k ") e −0.4 m = 0.209 1.225(1.225m)7.25−1 −1.225m dm × ×e Γ(7.25) 9.12 Let X =
compression index X is N(µ , 0.16) Sample mean x = 1 (0.75 + 0.89 + 0.91 + 0.81) = 0.84 4 (a) From Equation 8.13, f’’(µ) = N (x, σ) n = N(0.84, 0.16/2) = N(0.84, 0.08) (b) f’(µ) = N(µ µ' , σ µ') = N(0.8, 0.2) L(µ) = N(0.84, 0.08) = N (x, σ) n From Equation 9.14, 9.15, µ µ'' = x(σ µ') 2 + µ µ' σ 2 / n (σ µ') 2 + σ 2 / n = 0.84 × (0.2) 2 + 0.8 ×

(0.08) 2 0.2 2 + 0.08 2 = 0.834 and, σμ = '' (σ μ') 2 (σ 2 / n) (σ μ') 2 + (σ 2 / n) = 0.2 2 × 0.08 2 = 0.0743 0.2 2 + 0.08 2 So, f’’(μ) = N(0.834, 0.0743) (c) P(μ < 0.95) = φ (0.95 − 0.834) = φ (1.56) = 0.94 0.0743 9.13 (a) T is N(μ , 10) Sample mean = t = 65 min. n=5 So the posterior distribution of μ , f ' ' (μ) = N (t , σ) = N (65, 10 n) = N (65,4.47) min. 5 (b) We know, f’(μ) = N(65,4.47) min. L(μ) = N (60, 10) = N(60, 3.16) min. 10 From Equation 8.14 and 8.15, μ μ'' = T σμ = '' T 65 × 3.162 + 60 × 4.47 2 = 61.7 min . 4.47 2 + 3.162 4.47 2 × 3.16 2 = 2.58 min . 4.47 2 + 3.16 2 f’’(μ T) = N(61.7, 2.58) min. (c) From Equation 9.17, fT(t) = N(61.7, 102 + 2.582) = N(61.7, 10.33) min. P(T > 80) = 1 − φ (80 − 61.7) = 1 − φ (1.77) = 0.0384 10.33 9.14 (a) The sample mean θ = sθ = 2 1 (32 o 04'+31o 59'+32 o 01'+32 o 05'+31o 57'+32 o 00') = 32 o 01' 6 1 {(3') 2 + (2') 2 + 0 + (4') 2 + (4') 2 + (1') 2 } = 9.2 6 −1 sθ = 3.03' = 0.05 o The actual value of the angle is N (32 o 01' , 0.05 o) 6 or, N(32.0167o, 0.0204o) The value of the angle would be 32.0167 ± 0.0204o or 32o01’ ± 1.224’ (b) The prior distribution of θ can be modeled as N(32o, 2’) or N(32o, 0.0333o). 9.14 and 9.15, the Bayesian estimate of the elevation is, 32 × 0.0204 2 + 32.0167 × 0.0333 2 θ'' = 0.0204 2 + 0.0333 2 = 32.0121o = 32o0.726’ and the corresponding standard error is, σ θ'' = 0.0204 2 · 0.0333 2 = 0.01740 o or 1.044' 2 2 0.0204 + 0.0333 Hence,1.044’ θ is 32o0.726’ ± 1.044’ Applying Eqs. 9.15 Assume, First measurement → L=N(2.15, σ) Second measurement → L=N(2.20, 2σ) and Third measurement → L=N(2.18, 3σ) Applying Eqs. 9.14 and 9.15 and considering First and Second measurements, L' ' = 2.20 × σ 2 + 2.15 × (2σ) 2 = 2.16km σ 2 + (2σ) 2 and, σ L'' = σ 2 × (2σ) 2 = 0.8944σ σ 2 + (2σ) 2 Now consider L’’ with the third measurement and apply Eq. 9.14 again, 2.16 × (3σ) 2 + 2.18 × (0.8944σ) 2 L' ' ' = = 2.1616km (3σ) 2 + (0.8944σ) 2 9.16 Assume the prior distribution of p to be a Beta distribution, then, E ' (p) = q' = 0.1 ---------- (1) q'+ r ' and Var ' (p) = a' r ' = 0.06 2 ---------(2) 2 (q'+ r ') (q '+ r '+1) From Equations 1 and 2, we get, q’ = 2.4 and r’ = 9 × 2.4 = 21.6 From Table 9.1, for a binomial basic random variable, q’’ = q’ + x = 2.4 + 1 = 3.4 v’’ = v’ + n-x = 21.6 + 12 – 1 = 32.6 So E’’(p) = 3.4 / (3.4 + 32.6) = 0.0945 Var ' ' (p) = 3.4 × 32.6 = 0.00231 (3.4 + 32.6) 2 (3.4 + 32.6 + 1) 9.17 μ is the parameter to be updated with the prior distribution parameter μμ' = 3 σ μ' = 0.2 × 3 = 0.6 (a) From the observation we get t = N(2 + 3 0.5 ,) = N (2.5,0.354) 2 2 From the relationship given in Table 9.1, 0.354 2 × 3 + 0.62 × 2.5 μμ = = 2.629 0.62 + 0.354 2 '' 0.3542 × 0.62 σμ = = 0.305 0.62 + 0.354 2 '' (b) From equation 9.17 the distribution of T is T = N (μμ'' , σ 2 + σ μ'') = N (2.629, 0.62 + 0.3052) = N (2.629,0.673) 2 P(T < 1) = P (T − 2.629 1 − 2.629 <) = Φ (−2.421) = 0.774% 0.673 0.673 9.18 (a) The parameter needs to be updated is p. It’s suitable to set a conjugate prior for the problem. Thus, suppose p is Beta distributed with parameter p’ and q’. With the information presented in the problem, we can get E '(p) = q' q'r' = 0.5 Var '(p) = = 0.05 2 q '+ r ' (q ' + r ') (q ' + r ' + 1) , Thus, q ' = r ' = 2 (b) Upon the observation the posterior distribution of p can be updated in terms of p and q as: q '' = q '+ 2 = 2 + 2 = 4 r '' = r '+ n − x = 2 + 1 − 2 = 1 f ''(p) = Γ(5) p3 = 4 p3 Γ(4) Γ(1) (c) Let ε =win the next two bids 1 1 0 0 p (ε) = ∫ P(ε | p) f ''(p)dp = ∫ p 2 i4 p 3dp = 2 3 9.19 Using conjugate distributions, we assume Gamma distribution as prior distribution of λ. From Table 9.1, for an exponential basic random variable, E ' (λ) = k' = 0.5; v' Var ' (λ) = k ' 0 .5 = = (0.5 × 0.2) 2 2 v' v' So, v' = 50 and, k' = 25 Now, v'' = v' + ∑x i = 50 + (1 + 1.5) = 52.5 k'' = k' + n = 25 + 2 =27 E''(λ) = k''/v'' = 27/52.5 = 0.514 Var''(λ) = k''/v''2 = 27/52.52 = 9.796 x 10-3 σ''(λ) = 0.099 δ λ'' = 0.099 = 0.193 0.514 9.20 λ is the parameter to be updated. Let X denote the crack length. The probability of crack length larger than 4 is: P(X > 4) = 1 − P (X < 4) = e −4 λ The probability of crack length smaller than 6 is: P(X < 6) = 1 − e −6 λ Likelihood function is L(λ) = λ e −3λ iλ e −5λ i(1 − e −6 λ)iλ e −4λ i e −4 λ iλ e −8λ = λ 4 e −24 λ (1 − e −6λ) non-informative prior is used, thus f ''(λ) ∝ L(λ) = λ 4 e −24 λ (1 − e −6 λ) ∫ +∞ 0 λ 4e −24λ (1 − e −6λ)dλ = 1/ 23415 thus f ''(λ) = 23415λ 4 e −24 λ (1 − e −6 λ) 9.21 (a) The occurrence rate of the tornado is estimated from the historical record as 0.1/year. The probability of x occurrences during time t (in years) is P(X = x) = (0.1t) x −0.1t e x! The probability that the tornado hits the town during the next 5 years is P(X > 0) = 1 − P(X = 0) = 1 − e −0.5 = 0.393 The prior distribution parameter of mean maximum wind speed is: μμ' = 145 + 175 = 160 2 μμ' − 145 μμ' − 145 15 ' = = 7.653 = 1.96 , thus σ μ = 1.96 1.96 σ μ' The updated the distribution parameter of mean maximum wind speed is μμ'' = 160 × 152 + 175 × 7.6532 2 = 164.854 152 2 + 7.653 2 152 × 7.6532 = 6.143 σ μ'' = 22 15 2 + 7.653 2 From equation 9.17, the updated the distribution of maximum wind speed is v = N (μμ'' , σ μ'' + σ 2) = N (164.854, 152 + 6.1432) = N (164.854,16.209) 2 P(v > 220) = 1 − P(v ≤ 220) = 1 − P(T − 164.854 220 − 164.854 <) = 1 − Φ (3.402) = 0.033% 16.209 16.209 (3) The probability that the house will be damaged in the next 5 years is P(v > 120, x > 0) = P (v > 220) P(x > 0) = 0.876 × 0.393 = 0.344 where P(v > 120) = 1 − P (v ≤ 120) = 1 − P(T − 161.376 120 − 161.376 <) = 1 − Φ (−1.157) = 0.876 35.756 35.756 P(x > 0) = 1 − P(x = 0) = 1 − e −0.1×5 = 0.393 The probability that the house will not be damaged is 1 − 0.344 = 0.656 9.22 h is the parameter to be updated. (a) 10 10 0 0 E (x) = ∫ E (x | h) f (h)dh = ∫ 0.5hi0.003h 2 = 3.75 where h h 0 0 E (x | h) = ∫ xf (x | h)dx = ∫ x dh = 0.5h h (b) (i) the range of h is 4 0.83) = 1 − 0.7782 = 0.395 So the second requirement is more stringent. 9.24 (1) Prior Non-informative prior is used as (P.M. Lee. Bayesian Statistics: An Introduction. Edward Arnold, 1989) f '(μT , σ T2) ∝ 1 σ T2 (2) Likelihood , tn | μT , σ T) p(t1 , t2 , ⎡ (t1 − μT) 2 ⎤ ⎡ (t2 − μT) 2 ⎤ ⎡ (tn − μT) 2 ⎤ 1 1 = exp ⎢ − i exp ⎢ − i i exp ⎢ − 2σ T2 ⎥⎦ 2πσ T 2σ T2 ⎥⎦ 2σ T2 ⎥⎦ 2πσ T 2πσ T ⎡ ⎤ ⎡ ⎤ 1 = = (⎡ ∑ (ti − μT) 2 ⎤ exp ⎢ − ⎥ n 2σ T2 2πσ T ⎥⎦ (⎡ (n − 1) s 2 + n (t − μ) 2 ⎤ exp n ⎢ − ⎥ 2σ T2 ⎥⎦ 2πσ T 1) 1) where s 2 = 1 n (ti − t) 2 ∑ n − 1 i =1 (3) Joint posterior is: ⎡ (n − 1) s 2 + n (t − μ) 2 ⎤ f ''(μT , σ T2) ∝ σ T− n −2 exp ⎢ − ⎥ 2σ T2 ⎥⎦ (4) Marginal posterior of σ T2 f ''(σ T2) = ∫ f ''(μT , σ T2)d μT ∝ ∫σ − n −2 T ∝ (σ T2) ⎡ (n − 1) s 2 + n (y − μ) 2 ⎤ exp ⎢ − ⎥ d μT 2σ T2 ⎥⎦ − n −1 2 ⎡ (n − 1) s 2 ⎤ exp ⎢ − 2σ T2 ⎥⎦ Which is a scaled inverse- χ 2 density ( ) So σ T2 '' = Inv − χ 2 (n − 1, s 2) (5) Marginal posterior of μT Similarly, the posterior of μT can be written as f ''(μT) = ∫ f ''(μT , σ T2)dσ T2 ⎡ (n − 1) s 2 + n (t − μ) 2 ⎤ 2 ∝ ∫ σ T− n −2 exp ⎢ − ⎥ dσ T 2σ T2 ⎥⎦ ∞ ∝ A−n 2 ∫ z (n −2) 2 exp(− z)dz 0 ∝ ⎡⎢(n − 1) s 2 + n (μT − t) 2 ⎤⎥ ⎡ n(μT − t) 2 ⎤ ∝ ⎢1 + (n − 1) s 2 ⎥⎦ ⎢ −n / 2 −n / 2 which is the tn −1 (t , s 2 n) density. Or it can be written as μT'' − t s n = tn −1 Where z = A 2σ 2 T , A = (n − 1) s 2 + n(μT − t) 2 (6) We need both sample mean and sample standard deviation in this problem. From the information available in the problem, only the first set of data with 5 flight time has both sample mean and sample standard deviation. So only this set of data will be used. s = 10 , n = 5 , t = 65 , S = (n − 1) s 2 = (5 − 1) × 102 = 400 μT'' = tn −1 (t , s 2 n) = t4 (65, 4.472) E (μT'') = 65 , Var (μT'') = (σ) 2 " T 4 × 4.4722 = 39.998 4−2 = Inv − χ 2 (n − 1, s 2) = Inv − χ 2 (4,100) " 4 × 100 = 200 E ⎡⎢ (σ T2) ⎤⎥ = ⎣ ⎦ 4−2 " 2 × 42 Var ⎡⎢ (σ T2) ⎤⎥ = × 100 = ∞ ⎣ ⎦ (4 − 2) 2 (4 − 4) 9.25 Let x denotes the rated value, and y denotes the actual value. Based on 9.24 to 9.27, α = 0.512 , β = 0.751 , σ 2 = 1.732 , s x2 = 34.167 From equation 9.26, E (Y | x) = 0.512 + 0.751x The variance can be calculated by equation 9.34 as Var (Y | x) = 6 −1 ⎡ 1 ⎡ 6 (x − 24.167) 2 ⎤ ⎤ 2 × ⎨1 + ⎢1 + ⎥ ⎬ × 1.732 = 3.368 + 0.0169(x − 24.167) ⎢ 6 − 3 ⎣ 6 ⎣ 6 − 1 34.167 ⎦ ⎦ E (Y | x = 24) = 0.512 + 0.751 × 24 = 18.536 Var (Y | x = 4) = 3.368 + 0.0169(24 − 24.167) 2 = 3.368 P(Y < 18 | x = 24) = Φ (18 − 18.536) = 0.385 3.368 if the value of the basic scatter σ 2 is equal to 1.73, the variance of Y at x = 24 becomes 1.73. Then P(Y < 18 | x = 24) = Φ (18 − 18.536) = 0.34 1.73 10.1 (a) Apply Equation 10.2 g(0.10) ≤ 0.05 For n = 20, and r = 1 ⎛ 20 ⎞ ⎛ 20 ⎞ ⎜⎜(0.10) 0 (0.90) 20 + ⎜ ⎜(0.10) 1 (0.90) 19 ⎝1⎠ ⎝0⎠ g(0.10) = ⎜ = 0.1216 + 0.2702 = 0.05 The welds should not be accepted. r (b) g(0.10) = ⎛ n⎞ ∑ ⎜⎜ x ⎜⎜(0.03) x =0 x (0.97) n − x = 0.90 x (0.90) n − x = 0.05 ⎝ ⎠ and from part (a) r g(0.10) = ⎛n⎞ ∑ ⎜⎜ x ⎜⎜(0.10) x =0 ⎝ ⎠ From trial and error, and using Table of Cumulative Binomial Probabilities, n = 105, r = 5. (c) AOQ = pg(p) Here, n = 25 and r = 1 So, 1 AOQ = p ⎛ 25 ⎞ ∑⎜ x ⎜ p x =0 ⎝ ⎠ x (1 − p) 25− x = p · (1 − p) 25 + p 2 × 25 × (1 − p) 24 This is plotted in Fig 10.1. From the graph, AOQL = 0.0332 Fig. 10.1 AOQ Curve 10.2 P(SO2 < 0.1 unit) ≥ 0.90 with 95% confidence i.e. to achieve a reliability of 0.9 with 95% confidence n= ln(0.05) = 28.4 ≈ 29days ln(0.90) 10.3 (a) Rn = 1 – C Here, n = 10 and R = 1 - 0.15 = 0.85 So, C = 1 - Rn = 1 - (0.85)10 = 0.8031 (b) C = 0.99, R = 0.85, n = ln(1 − C) ln(0.01) = = 28.3 ≈ 29 ln R ln(0.85) So 19 additional borings should be made. 10.4 n= ln(1 − C) ln R Here, C = 0.95 and R = 0.99 So, n= ln(1 − 0.95) = 298 ln(0.99) 298 consecutive successful starts would be required to meet the standard. 10.5 ⎛ n⎞ r g(0.05) = ∑ ⎜⎜ x ⎜⎜(0.05) x =0 ⎝ ⎠ (0.95) n − x = 0.94 and, r g(0.20) = ⎛n⎞ ∑ ⎜⎜ x ⎜⎜(0.20) x =0 ⎝ ⎠ x (0.80) n − x = 0.10 The values of n and r are to be determined by trial and error. Alternatively, from fig. 10.1e, n = 30 and r = 3 approximately satisfy the above two equations. 10.6 (a) p = fraction defective = 0.30 n = 5, r = 0 From Equation 10.2, ⎛ 5⎞ g(0.3) = ⎜⎜ ⎜⎜(0.3) 0 (0.7) 5 = (0.7) 5 = 0.168 0 ⎝ ⎠ = consumer risk ⎛ 5⎞ (b) g(0.1) = ⎜⎜ ⎜⎜(0.1) 0 (0.9) 5 = 0.59 0 ⎝ ⎠ ∴ P(rejection) = 1 - g(0.1) = 0.41 10.7 (a) α = Producer’s risks = P (X < L μ a) = Φ (L − μa σ) = Φ(118 − 120) = Φ (−1.58) = 0.057 4 n 10 L − μt 118 − 114 β = Consumer’s risks = P (X > L μ t) = 1 − Φ ( ) = 1 − Φ( ) = 1 − Φ (3.16) = 0.000792 σ 4 n (b) α = 0.10 = Φ (L − 120) 4 n L − 114 β = 0.05 = 1 − Φ ( ) 4 (or or 10 L − 120) n = −1.28 4 n(L − 114) = 1.64 4 n ∴ L − 120 − 1.28 = → L = 117.37 L − 114 1.64 From this, n= 3.8, so take n = 4.0 and L = 117.34 lb/ft3 (c) Using equations 10.10 and 10.11, n (L − 120) = −t 0.10, n −1 4 and n (L − 114) = t 0.05, n −1 4 After a trial-and-error procedure, n is found to be 6 and the corresponding value of L becomes 117.59 lb/ft3 or 117.29 lb/ft3. So take the average L in order to be fair to both consumer and producer and L = 117.44 lb/ft3. 10.8 α = 0.10, β = 0.10 Pa = 0.02, Pt = 0.10 The required sample size n is, 2 ⎡ Φ −1 (1 − β) − Φ −1 (α) ⎤ ⎡ Φ −1 (0.90) − Φ −1 (0.10) ⎤ = n = ⎢ −1 ⎥ ⎢ −1 ⎥ −1 ⎢ Φ (0.10) − Φ (0.02) ⎥ ⎢ Φ (Pt) − Φ (Pa) ⎥ 2 2 2 ⎡ 1.28 − (−1.28) ⎤ ⎡ 2.56 ⎤ =⎢ ⎢ ⎥ = 10.77 ≈ 11 ⎢ 0.78 ⎥ ⎢ − 1.28 − (−2.06) ⎥ The corresponding tolerable fraction defective M is ⎡ −1 ⎡ −1 Φ −1 (α) ⎤ Φ −1 (0.10) ⎤ M = Φ ⎢ Φ (Pa) − ⎥ = Φ ⎢ Φ (0.02) − ⎥ n ⎥ 11 ⎢ ⎢ ⎡ 1.28 ⎤ = Φ ⎢ − 2.06 + ⎥ = Φ (−1.674) = 0.047 11 ⎢ 10.9 μa = 0.90, μt = 0.80, α = 0.05, β = 0.05, σ = 0.04 Φ(L − 0.90) = 0.05 0.04 n(or L − 0.90) = −1.64 0.04 n and 1 − Φ (L − 0.80) = 0.05 0.04 or n(L − 0.80) = 1.64 0.04 n Solving the above two equations, n = 2 and L = 0.85.

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